

ESO - BACHILLERATO

ECUACIONES

<https://aprendeconmigomelon.com>

10 de diciembre de 2024



IÑIGO ZUNZUNEGUI MONTEERRUBIO

En este libro he condensado todos los tipos de ecuaciones que te vas a encontrar en los últimos cursos de la ESO y en 1º de Bachillerato. Podrás practicar ecuaciones algebraicas, irracionales, polinómicas, exponenciales y logarítmicas, así como sistemas de ecuaciones lineales, no lineales, exponenciales y logarítmicos. Todas están resueltas y espero que te resultan útiles melón ;)

Índice general

1. Ecuaciones lineales	3
1.1. Sistemas de ecuaciones lineales	3
2. Ecuaciones polinómicas	9
2.1. Ecuaciones de 2º grado	9
2.2. Ecuaciones bicuadradas y bicúbicas	13
2.3. Ecuaciones Polinómicas de grado superior	15
3. Ecuaciones con fracciones	21
3.1. Ecuaciones racionales	21
4. Ecuaciones irracionales	33
4.1. Ecuaciones irracionales	33
5. Ecuaciones no lineales	43
5.1. Sistemas de ecuaciones no lineales	43
6. Exponenciales	55
6.1. Ecuaciones exponenciales	55
6.2. Sistemas de ecuaciones exponenciales	67
7. Logaritmos	73
7.1. Definición de logaritmo	73
7.1.1. Calcula el valor de los siguientes logaritmos:	73
7.1.2. Halla el valor de x en los siguientes logaritmos:	76
7.2. Ecuaciones logarítmicas	78
7.3. Sistemas de ecuaciones logarítmicas	96

1.1. Sistemas de ecuaciones lineales

$$\textcircled{1} \begin{cases} 10x - 9y = -84 \\ -2x + y = 12 \end{cases}$$

$$\textcircled{2} \begin{cases} 5x - y = 12 \\ x + 3y = 12 \end{cases}$$

$$\textcircled{3} \begin{cases} 3x + y = 10 \\ x + 3y = 6 \end{cases}$$

$$\textcircled{4} \begin{cases} 3x + y = 17 \\ -2x + 3y = 7 \end{cases}$$

$$\textcircled{5} \begin{cases} 2x + 3y = 17 \\ 3x + 2y = 18 \end{cases}$$

$$\textcircled{6} \begin{cases} -2x + 5y = 22 \\ 3x - 6y = -27 \end{cases}$$

$$\textcircled{7} \begin{cases} 3x - 4y = 26 \\ x - 8y = 22 \end{cases}$$

$$\textcircled{8} \begin{cases} 5x - 4y = 28 \\ -3x + y = 7 \end{cases}$$

$$\textcircled{9} \begin{cases} 3x + 4y = 19 \\ 7x - 2y = -1 \end{cases}$$

$$\textcircled{10} \begin{cases} 4x + 7y = 5 \\ 6x - 2y = 20 \end{cases}$$

$$\textcircled{11} \begin{cases} 4x + 7y = -56 \\ -2x - 5y = 40 \end{cases}$$

$$\textcircled{12} \begin{cases} 7x - 14y = -5 \\ -7x + 21y = 9 \end{cases}$$

$$\textcircled{13} \begin{cases} x + y = -2 \\ x - y = 0 \end{cases}$$

$$\textcircled{14} \begin{cases} 5x + 3y = 31 \\ 4x + 2y = 24 \end{cases}$$

$$\textcircled{15} \begin{cases} 3x + 9y = 4 \\ 4x - 6y = 1 \end{cases}$$

$$\textcircled{16} \begin{cases} 3x + 11y = 67 \\ 5x - 3y = 5 \end{cases}$$

$$\textcircled{17} \begin{cases} 3x + 2y = -7 \\ 6x + 16y = -2 \end{cases}$$

$$\textcircled{18} \begin{cases} 3x + 6y = 39 \\ 7x - 3y = 57 \end{cases}$$

1.1. Sistemas de ecuaciones lineales

1

$$\begin{cases} 10x - 9y = -84 & \Rightarrow 10x - 9 \cdot (12 + 2x) = -84 \\ & \Rightarrow 10x - 108 - 18x = -84 \Rightarrow -8x = 24 \Rightarrow \boxed{x = -3} \\ -2x + y = 12 & \Rightarrow y = 12 + 2x \end{cases}$$

$$\Rightarrow y = 12 + 2x = 12 + 2 \cdot (-3) \Rightarrow \boxed{y = 6}$$

2

$$\begin{cases} 5x - y = 12 & \Rightarrow y = 5x - 12 \\ x + 3y = 12 & \Rightarrow x + 3 \cdot (5x - 12) = 12 \Rightarrow x + 15x - 36 = 12 \Rightarrow \boxed{x = 3} \end{cases}$$

$$\Rightarrow y = 5x - 12 = 5 \cdot 3 - 12 \Rightarrow \boxed{y = 3}$$

3

$$\begin{cases} 3x + y = 10 & \Rightarrow y = 10 - 3x \\ x + 3y = 6 & \Rightarrow x + 3 \cdot (10 - 3x) = 6 \Rightarrow x + 30 - 9x = 6 \Rightarrow \boxed{x = 3} \end{cases}$$

$$\Rightarrow y = 10 - 3x = 10 - 3 \cdot 3 \Rightarrow \boxed{y = 1}$$

4

$$\begin{cases} 3x + y = 17 & \Rightarrow y = 17 - 3x \\ -2x + 3y = 7 & \Rightarrow -2x + 3 \cdot (17 - 3x) = 7 \Rightarrow -2x + 51 - 9x = 7 \Rightarrow \boxed{x = 4} \end{cases}$$

$$\Rightarrow y = 17 - 3x = 17 - 3 \cdot 4 \Rightarrow \boxed{y = 5}$$

5

$$\begin{cases} 2x + 3y = 17 & \Rightarrow y = \frac{17 - 2x}{3} \\ 3x + 2y = 18 & \Rightarrow 3x + 2 \cdot \frac{17 - 2x}{3} = 18 \Rightarrow 9x + 34 - 4x = 54 \end{cases}$$

$$\Rightarrow 5x = 20 \Rightarrow \boxed{x = 4}$$

$$\Rightarrow y = \frac{17 - 2x}{3} = \frac{17 - 2 \cdot 4}{3} \Rightarrow \boxed{y = 3}$$

6

$$\begin{cases} -2x + 5y = 22 & \Rightarrow y = \frac{22 + 2x}{5} \\ 3x - 6y = -27 & \Rightarrow 3x - 6 \cdot \frac{22 + 2x}{5} = -27 \Rightarrow 15x - 132 - 12x = -135 \end{cases}$$

$$\Rightarrow 3x = -3 \Rightarrow \boxed{x = -1}$$

$$\Rightarrow y = \frac{22 + 2 \cdot (-1)}{5} \Rightarrow \boxed{y = 4}$$

7

$$\begin{cases} 3x - 4y = 26 & \Rightarrow x = \frac{26 + 4y}{3} \\ x - 8y = 22 & \Rightarrow x = 22 + 8y \end{cases} \Rightarrow \frac{26 + 4y}{3} = 22 + 8y \Rightarrow 26 + 4y = 66 + 24y$$

$$\Rightarrow 20y = -40 \Rightarrow \boxed{y = -2}$$

$$\Rightarrow x = \frac{26 + 4y}{3} = \frac{26 + 4 \cdot (-2)}{3} \Rightarrow \boxed{x = 6}$$

8

$$\begin{cases} 5x - 4y = 28 & \Rightarrow y = \frac{5x - 28}{4} \\ -3x + y = 7 & \Rightarrow y = 7 + 3x \end{cases} \Rightarrow \frac{5x - 28}{4} = 7 + 3x \Rightarrow 5x - 28 = 28 + 12x$$

$$\Rightarrow 7x = -56 \Rightarrow \boxed{x = -8}$$

$$\Rightarrow y = 7 + 3x = 7 + 3 \cdot (-8) \Rightarrow \boxed{y = -17}$$

9

$$\begin{cases} 3x + 4y = 19 & \Rightarrow x = \frac{19 - 4y}{3} \\ 7x - 2y = -1 & \Rightarrow x = \frac{2y - 1}{7} \end{cases} \Rightarrow \frac{19 - 4y}{3} = \frac{2y - 1}{7} \Rightarrow 133 - 28y = 6y - 3$$

$$\Rightarrow 34y = 136 \Rightarrow \boxed{y = 4}$$

$$\Rightarrow x = \frac{19 - 4y}{3} = \frac{19 - 4 \cdot 4}{3} \Rightarrow \boxed{x = 1}$$

10

$$\begin{cases} 4x + 7y = 5 & \Rightarrow y = \frac{5 - 4x}{7} \\ 6x - 2y = 20 & \Rightarrow y = \frac{6x - 20}{2} \end{cases} \Rightarrow \frac{5 - 4x}{7} = \frac{6x - 20}{2} \Rightarrow 10 - 8x = 42x - 140$$

$$\Rightarrow 50x = 150 \Rightarrow \boxed{x = 3}$$

$$\Rightarrow y = \frac{5 - 4x}{7} = \frac{5 - 4 \cdot 3}{7} \Rightarrow \boxed{y = -1}$$

1.1. Sistemas de ecuaciones lineales

11

$$\begin{cases} 4x + 7y = -56 & \Rightarrow y = \frac{-56 - 4x}{7} \\ -2x - 5y = 40 & \Rightarrow y = \frac{-2x - 40}{5} \end{cases} \Rightarrow \frac{-56 - 4x}{7} = \frac{-2x - 40}{5}$$

$$\Rightarrow -280 - 20x = -14x - 280 \Rightarrow 6x = 0 \Rightarrow \boxed{x = 0}$$

$$\Rightarrow y = \frac{-56 - 4x}{7} = \frac{-56}{7} \Rightarrow \boxed{y = -8}$$

12

$$\begin{cases} 7x - 14y = -42 & \Rightarrow x = \frac{14y - 42}{7} \\ -7x + 21y = 70 & \Rightarrow x = \frac{21y - 70}{7} \end{cases} \Rightarrow \frac{14y - 42}{7} = \frac{21y - 70}{7} \Rightarrow 14y - 42 = 21y - 70$$

$$\Rightarrow 7y = 28 \Rightarrow \boxed{y = 4}$$

$$\Rightarrow x = \frac{14y - 42}{7} = \frac{14 \cdot 4 - 42}{7} \Rightarrow \boxed{x = 2}$$

13

$$\begin{cases} x + y = -2 \\ x - y = 0 \end{cases}$$

$$\begin{array}{r} x + y = -2 \\ x - y = 0 \\ \hline 2x = -2 \end{array} \Rightarrow \boxed{x = -1}$$

$$x + y = -2 \Rightarrow -1 + y = -2 \Rightarrow \boxed{y = -1}$$

14

$$\begin{cases} 5x + 3y = 31 \\ 4x + 2y = 24 \end{cases}$$

$$5x + 3y = 31 \xrightarrow{\times 2} 10x + 6y = 62$$

$$4x + 2y = 24 \xrightarrow{\times (-3)} -12x - 6y = -72$$

$$\begin{array}{r} 10x + 6y = 62 \\ -12x - 6y = -72 \\ \hline -2x = -10 \end{array} \Rightarrow \boxed{x = 5}$$

$$5x + 3y = 31 \Rightarrow 5 \cdot 5 + 3y = 31 \Rightarrow \boxed{y = 2}$$

15

$$\begin{cases} 3x + 9y = 4 \\ 4x - 6y = 1 \end{cases}$$

$$3x + 9y = 4 \xrightarrow{\times (-4)} -12x - 36y = -16$$

$$4x - 6y = 1 \xrightarrow{\times 3} 12x - 18y = 3$$

$$\begin{array}{r} -12x - 36y = -16 \\ 12x - 18y = 3 \\ \hline -54y = -13 \end{array} \Rightarrow \boxed{y = 13/54}$$

$$3x + 9y = 4 \Rightarrow 3x + 9 \cdot \frac{13}{54} = 4 \Rightarrow \boxed{x = 11/18}$$

$$16 \quad \begin{cases} 3x + 11y = 67 \\ 5x - 3y = 5 \end{cases}$$

$$\begin{array}{rcl} 3x + 11y = 67 & \xrightarrow{\times 5} & \cancel{15x} + 55y = 335 \\ 5x - 3y = 5 & \xrightarrow{\times (-3)} & \underline{\cancel{-15x} + 9y = -15} \\ & & 64y = 320 \implies \boxed{y = 5} \end{array}$$

$$5x - 3y = 5 \implies 5x - 3 \cdot 5 = 5 \implies \boxed{x = 4}$$

$$17 \quad \begin{cases} 3x + 2y = -7 \\ 6x + 16y = -2 \end{cases}$$

$$\begin{array}{rcl} 3x + 2y = -7 & \xrightarrow{\times (-2)} & \cancel{-6x} - 4y = 14 \\ 6x + 16y = -2 & \longrightarrow & \underline{\cancel{6x} + 16y = -2} \\ & & 12y = 12 \implies \boxed{y = 1} \end{array}$$

$$3x + 2y = -7 \implies 3x + 2 \cdot 1 = -7 \implies \boxed{x = -3}$$

$$18 \quad \begin{cases} 3x + 6y = 39 \\ 7x - 3y = 57 \end{cases}$$

$$\begin{array}{rcl} 3x + 6y = 39 & \longrightarrow & 3x + \cancel{6y} = 39 \\ 7x - 3y = 57 & \xrightarrow{\times 2} & \underline{14x - \cancel{6y} = 114} \\ & & 17x = 153 \implies \boxed{x = 9} \end{array}$$

$$3x + 6y = 39 \implies 3 \cdot 9 + 6y = 39 \implies \boxed{y = 2}$$

Ecuaciones polinómicas

2.1. Ecuaciones de 2º grado

① $(x - 2) \cdot (x - 3) = 0$

② $x \cdot (2x - 4) = 0$

③ $(x + 1) \cdot (2x - 1) = 0$

④ $(x - 2)^2 = 0$

⑤ $7 \cdot (2x - 6) \cdot (x + 3) = 0$

⑥ $(x - 4) \cdot (x + 3) = 0$

⑦ $3x^2 - 27 = 0$

⑧ $2x^2 - 4x = 0$

⑨ $x^2 = 16$

⑩ $9x^2 = 4$

⑪ $\frac{2x^2}{3} - 6 = 0$

⑫ $2x^2 - 32 = 0$

⑬ $25x^2 - 9 = 0$

⑭ $6x^2 - 2x = 0$

⑮ $x^2 \cdot (x - 3) \cdot (3x + 4) = 0$

⑯ $x^3 \cdot (x - 1)^2 = 0$

⑰ $5x^3 \cdot (x^2 - 9) \cdot (x^2 - 2) = 0$

⑱ $3x^2 - 3x - 18 = 0$

⑲ $-2x^2 + 3x + 5 = 0$

⑳ $4x^2 - 12x + 9 = 0$

㉑ $3x^2 - 5x + 8 = 0$

㉒ $2x \cdot (3x - 5) + 7x \cdot (1 - x) = -10$

㉓ $3x^2 + x \cdot (5 - 3x) - 42 = 6 \cdot (x - 7)$

㉔ $2x \cdot (5x - 1) - 6x^2 + 2 \cdot (x - 5) = 0$

㉕ $3x + 5x \cdot (x - 1) + 8x - 7 = -8$

㉖ $-4x \cdot (x - 2) + 4 \cdot (x + 2) = 16$

㉗ $x \cdot (-x + 2) - 3x \cdot (x + 2) = -8$

2.1. Ecuaciones de 2º grado

$$\textcircled{1} \quad (x-2) \cdot (x-3) = 0 \implies \begin{cases} x-2=0 \implies \boxed{x=2} \\ x-3=0 \implies \boxed{x=3} \end{cases}$$

$$\textcircled{2} \quad x \cdot (2x-4) = 0 \implies \begin{cases} \boxed{x=0} \\ 2x-4=0 \implies \boxed{x=2} \end{cases}$$

$$\textcircled{3} \quad (x+1) \cdot (2x-1) = 0 \implies \begin{cases} x+1=0 \implies \boxed{x=-1} \\ 2x-1=0 \implies \boxed{x=1/2} \end{cases}$$

$$\textcircled{4} \quad (x-2)^2 = 0 \implies x-2=0 \implies \boxed{x=2}$$

$$\textcircled{5} \quad 7 \cdot (2x-6) \cdot (x+3) = 0 \implies \begin{cases} 2x-6=0 \implies \boxed{x=3} \\ x+3=0 \implies \boxed{x=-3} \end{cases}$$

$$\textcircled{6} \quad (x-4) \cdot (x+3) = 0 \implies \begin{cases} x-4=0 \implies \boxed{x=4} \\ x+3=0 \implies \boxed{x=-3} \end{cases}$$

$$\textcircled{7} \quad 3x^2 - 27 = 0 \implies 3x^2 = 27 \implies x^2 = 9 \implies \boxed{x = \pm 3}$$

$$\textcircled{8} \quad 2x^2 - 4x = 0 \implies 2x \cdot (x-2) = 0 \implies \begin{cases} 2x=0 \implies \boxed{x=0} \\ x-2=0 \implies \boxed{x=2} \end{cases}$$

$$\textcircled{9} \quad x^2 = 16 \implies \boxed{x = \pm 4}$$

$$\textcircled{10} \quad 9x^2 = 4 \implies x^2 = \frac{4}{9} \implies \boxed{x = \pm 2/3}$$

$$\textcircled{11} \quad \frac{2x^2}{3} - 6 = 0 \implies \frac{2x^2}{3} = 6 \implies 2x^2 = 18 \implies x^2 = 9 \implies \boxed{x = \pm 3}$$

$$\textcircled{12} \quad 2x^2 - 32 = 0 \implies 2x^2 = 32 \implies x^2 = 16 \implies \boxed{x = \pm 4}$$

$$\textcircled{13} \quad 25x^2 - 9 = 0 \implies 25x^2 = 9 \implies x^2 = \frac{9}{25} \implies \boxed{x = \pm 3/5}$$

$$\textcircled{14} \quad 6x^2 - 2x = 0 \implies 2x \cdot (3x-1) = 0 \implies \begin{cases} 2x=0 \implies \boxed{x=0} \\ 3x-1=0 \implies \boxed{x=1/3} \end{cases}$$

$$\textcircled{15} \quad x^2 \cdot (x-3) \cdot (3x+4) = 0 \implies \begin{cases} x^2=0 \implies \boxed{x=0} \\ x-3=0 \implies \boxed{x=3} \\ 3x+4=0 \implies \boxed{x=-4/3} \end{cases}$$

$$\textcircled{16} \quad x^3 \cdot (x-1)^2 = 0 \implies \begin{cases} x^3=0 \implies \boxed{x=0} \\ (x-1)^2=0 \implies x-1=0 \implies \boxed{x=1} \end{cases}$$

$$(17) \quad 5x^3 \cdot (x^2 - 9) \cdot (x^2 - 2) = 0 \implies \begin{cases} 5x^3 = 0 \implies \boxed{x = 0} \\ x^2 - 9 = 0 \implies x^2 = 9 \implies \boxed{x = \pm 3} \\ x^2 - 2 = 0 \implies x^2 = 2 \implies \boxed{x = \pm \sqrt{2}} \end{cases}$$

$$(18) \quad 3x^2 - 3x - 18 = 0$$

$$\begin{cases} a = 3 \\ b = -3 \\ c = -18 \end{cases} \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 + 4 \cdot 3 \cdot 18}}{2 \cdot 3} = \frac{3 \pm \sqrt{225}}{6}$$

$$= \frac{3 \pm 15}{6} \implies \boxed{\begin{matrix} x_1 = 3 \\ x_2 = -2 \end{matrix}}$$

$$(19) \quad -2x^2 + 3x + 5 = 0$$

$$\begin{cases} a = -2 \\ b = 3 \\ c = 5 \end{cases} \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{9 + 4 \cdot 2 \cdot 5}}{2 \cdot (-2)} = \frac{-3 \pm \sqrt{49}}{-4}$$

$$= \frac{-3 \pm 7}{-4} \implies \boxed{\begin{matrix} x_1 = -1 \\ x_2 = 5/2 \end{matrix}}$$

$$(20) \quad 4x^2 - 12x + 9 = 0$$

$$\begin{cases} a = 4 \\ b = -12 \\ c = 9 \end{cases} \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{12 \pm \sqrt{144 - 4 \cdot 4 \cdot 9}}{2 \cdot 4} = \frac{12 \pm \sqrt{0}}{8}$$

$$= \frac{12 \pm 0}{8} \implies \boxed{x = 3/2 \text{ doble}}$$

$$(21) \quad 3x^2 - 5x + 8 = 0$$

$$\begin{cases} a = 3 \\ b = -5 \\ c = 8 \end{cases} \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{25 - 4 \cdot 3 \cdot 8}}{2 \cdot 3} = \frac{5 \pm \sqrt{-71}}{6} \Rightarrow \nexists \text{ Sol.}$$

$$(22) \quad 2x \cdot (3x - 5) + 7x \cdot (1 - x) = -10$$

$$6x^2 - 10x + 7x - 7x^2 = -10 \implies -x^2 - 3x = -10$$

$$\implies x^2 + 3x - 10 = 0 \quad \text{EC. COMPLETA}$$

$$\begin{cases} a = 1 \\ b = 3 \\ c = -10 \end{cases} \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{9 + 4 \cdot 1 \cdot 10}}{2 \cdot 1} = \frac{-3 \pm \sqrt{49}}{2}$$

$$= \frac{-3 \pm 7}{2} \implies \boxed{\begin{matrix} x_1 = 2 \\ x_2 = -5 \end{matrix}}$$

2.1. Ecuaciones de 2º grado

23) $3x^2 + x \cdot (5 - 3x) - 42 = 6 \cdot (x - 7)$

$$3x^2 + 5x - 3x^2 - 42 = 6x - 42 \implies 5x = 6x \implies \boxed{x = 0} \quad \text{EC. LINEAL}$$

24) $2x \cdot (5x - 1) - 6x^2 + 2 \cdot (x - 5) = 0$

$$10x^2 - 2x - 6x^2 + 2x - 10 = 0 \implies 4x^2 - 10 = 0 \quad \text{EC. INOMPLETA}$$

$$4x^2 = 10 \implies x^2 = \frac{5}{2} \implies \boxed{x = \pm\sqrt{5/2}}$$

25) $3x + 5x \cdot (x - 1) + 8x - 7 = -8$

$$3x + 5x^2 - 5x + 8x - 7 = -8 \implies 5x^2 + 6x + 1 = 0 \quad \text{EC. COMPLETA}$$

$$\begin{cases} a = 5 \\ b = 6 \\ c = 10 \end{cases} \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{36 - 4 \cdot 5 \cdot 1}}{2 \cdot 5} = \frac{-6 \pm \sqrt{16}}{10}$$

$$= \frac{-6 \pm 4}{10} \implies \boxed{\begin{matrix} x_1 = -1/5 \\ x_2 = -1 \end{matrix}}$$

26) $-4x \cdot (x - 2) + 4 \cdot (x + 2) = 16$

$$-4x^2 + 8x + 4x + 8 = 16 \implies 4x^2 - 12x + 8 = 0$$

$$\implies x^2 - 3x + 2 = 0 \quad \text{EC. COMPLETA}$$

$$\begin{cases} a = 1 \\ b = -3 \\ c = 2 \end{cases} \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 - 4 \cdot 1 \cdot 2}}{2 \cdot 1} = \frac{3 \pm \sqrt{1}}{2}$$

$$= \frac{3 \pm 1}{2} \implies \boxed{\begin{matrix} x_1 = 2 \\ x_2 = 1 \end{matrix}}$$

27) $x \cdot (-x + 2) - 3x \cdot (x + 2) = -8$

$$-x^2 + 2x - 3x^2 - 6x = -8 \implies 4x^2 + 4x - 8 = 0$$

$$\implies x^2 + x - 2 = 0 \quad \text{EC. COMPLETA}$$

$$\begin{cases} a = 1 \\ b = 1 \\ c = -2 \end{cases} \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1 + 4 \cdot 1 \cdot 2}}{2 \cdot 1} = \frac{-1 \pm \sqrt{9}}{2}$$

$$= \frac{-1 \pm 3}{2} \implies \boxed{\begin{matrix} x_1 = 1 \\ x_2 = -2 \end{matrix}}$$

2.2. Ecuaciones bicuadradas y bicúbicas

① $x^4 - 5x^2 + 4 = 0$

⑦ $4x^4 - 17x^2 + 4 = 0$

② $x^4 + 2x^2 - 3 = 0$

⑧ $9x^4 - 37x^2 + 4 = 0$

③ $x^6 - 9x^3 + 8 = 0$

⑨ $x^4 - 6x^2 - 27 = 0$

④ $x^6 - 26x^3 - 27 = 0$

⑩ $x^6 + 7x^3 - 8 = 0$

⑤ $6x^4 + 2x^2 - 8 = 0$

⑪ $x^4 - 2x^2 - 8 = 0$

⑥ $x^4 - 4x^2 = 0$

⑫ $x^6 + 28x^3 + 27 = 0$

[HTTPS://APRENDECONMIGOMELON.COM](https://aprendeconmigmelon.com)

2.2. Ecuaciones bicuadradas y bicúbicas

$$\textcircled{1} \quad x^4 - 5x^2 + 4 = 0 \xrightarrow[x^4=t^2]{x^2=t} t^2 - 5t + 4 = 0 \Rightarrow \begin{cases} t = 1 = x^2 \Rightarrow x = \pm 1 \\ t = 4 = x^2 \Rightarrow x = \pm 2 \end{cases}$$

$$\textcircled{2} \quad x^4 + 2x^2 - 3 = 0 \xrightarrow[x^4=t^2]{x^2=t} t^2 + 2t - 3 = 0 \Rightarrow \begin{cases} t = 1 = x^2 \Rightarrow x = \pm 1 \\ t = -3 = x^2 \Rightarrow \nexists \text{ Solución} \end{cases}$$

$$\textcircled{3} \quad x^6 - 9x^3 + 8 = 0 \xrightarrow[x^6=t^2]{x^3=t} t^2 - 9t + 8 = 0 \Rightarrow \begin{cases} t = 1 = x^3 \Rightarrow x = 1 \\ t = 8 = x^3 \Rightarrow x = 2 \end{cases}$$

$$\textcircled{4} \quad x^6 - 26x^3 - 27 = 0 \xrightarrow[x^6=t^2]{x^3=t} t^2 - 26t - 27 = 0 \Rightarrow \begin{cases} t = -1 = x^3 \Rightarrow x = -1 \\ t = 27 = x^3 \Rightarrow x = 3 \end{cases}$$

$$\textcircled{5} \quad 6x^4 + 2x^2 - 8 = 0 \xrightarrow[x^4=t^2]{x^2=t} 6t^2 + 2t - 8 = 0 \Rightarrow \begin{cases} t = 1 = x^2 \Rightarrow x = \pm 1 \\ t = -4/3 = x^2 \Rightarrow \nexists \text{ Solución} \end{cases}$$

$$\textcircled{6} \quad x^4 - 4x^2 = 0 \xrightarrow[x^4=t^2]{x^2=t} t^2 - 4t = 0 \Rightarrow t \cdot (t-4) = 0 \Rightarrow \begin{cases} t = 0 = x^2 \Rightarrow x = 0 \\ t - 4 = 0 \Rightarrow t = 4 = x^2 \Rightarrow x = \pm 2 \end{cases}$$

$$\textcircled{7} \quad 4x^4 - 17x^2 + 4 = 0 \xrightarrow[x^4=t^2]{x^2=t} 4t^2 - 17t + 4 = 0 \Rightarrow \begin{cases} t = 1/4 = x^2 \Rightarrow x = \pm 1/2 \\ t = 4 = x^2 \Rightarrow x = \pm 2 \end{cases}$$

$$\textcircled{8} \quad 9x^4 - 37x^2 + 4 = 0 \xrightarrow[x^4=t^2]{x^2=t} 9t^2 - 37t + 4 = 0 \Rightarrow \begin{cases} t = 1/9 = x^2 \Rightarrow x = \pm 1/3 \\ t = 4 = x^2 \Rightarrow x = \pm 2 \end{cases}$$

$$\textcircled{9} \quad x^4 - 6x^2 - 27 = 0 \xrightarrow[x^4=t^2]{x^2=t} t^2 - 6t - 27 = 0 \Rightarrow \begin{cases} t = -3 = x^2 \Rightarrow \nexists \text{ Solución} \\ t = 9 = x^2 \Rightarrow x = \pm 3 \end{cases}$$

$$\textcircled{10} \quad x^6 + 7x^3 - 8 = 0 \xrightarrow[x^6=t^2]{x^3=t} t^2 + 7t - 8 = 0 \Rightarrow \begin{cases} t = -8 = x^3 \Rightarrow x = -2 \\ t = 1 = x^3 \Rightarrow x = 1 \end{cases}$$

$$\textcircled{11} \quad x^4 - 2x^2 - 8 = 0 \xrightarrow[x^4=t^2]{x^2=t} t^2 - 2t - 8 = 0 \Rightarrow \begin{cases} t = -2 = x^2 \Rightarrow \nexists \text{ Solución} \\ t = 4 = x^2 \Rightarrow x = \pm 2 \end{cases}$$

$$\textcircled{12} \quad x^6 + 28x^3 + 27 = 0 \xrightarrow[x^6=t^2]{x^3=t} t^2 + 28t + 27 = 0 \Rightarrow \begin{cases} t = -1 = x^3 \Rightarrow x = -1 \\ t = -27 = x^3 \Rightarrow x = -3 \end{cases}$$

2.3. Ecuaciones Polinómicas de grado superior

① $x^3 - x^2 - 17x - 15 = 0$

② $x^3 - 2x^2 - 9x + 18 = 0$

③ $x^4 - x^3 - 13x^2 + x + 12 = 0$

④ $x^5 + x^4 - 8x^3 + 14x^2 - 8x = 0$

⑤ $4x^3 - 6x^2 - 6x + 4 = 0$

⑥ $x^4 - 6x^3 + 12x^2 - 8x = 0$

⑦ $6x^4 - 5x^3 - 14x^2 - x + 2 = 0$

⑧ $4x^5 - 12x^4 - 3x^3 + 10x^2 - 3x$

⑨ $x^3 - 4x = 0$

⑩ $x^3 - x^2 - 5x - 3 = 0$

⑪ $x^3 - 2x^2 - 5x + 6 = 0$

⑫ $x^4 - 9x^2 + 4x + 12 = 0$

⑬ $x^4 - 8x^3 + 14x^2 + 8x - 15 = 0$

⑭ $x^4 - 1$

⑮ $x^4 - x^3 - x^2 + x = 0$

⑯ $x^4 - 2x^3 - 13x^2 + 38x - 24 = 0$

⑰ $x^5 - 5x^4 + 7x^3 - 3x^2$

⑱ $3x^4 + 6x^3 + 6x^2 + 6x + 3 = 0$

⑲ $x^4 + 3x^3 + 4x^2 + 6x + 4$

2.3. Ecuaciones Polinómicas de grado superior

$$\textcircled{1} \quad x^3 - x^2 - 17x - 15 = 0 \xRightarrow{\textcircled{\bullet}} (x+1) \cdot (x^2 - 2x - 15) \Rightarrow \begin{cases} x + 1 = 0 \Rightarrow \boxed{x = -1} \\ x^2 - 2x - 15 = 0 \Rightarrow \begin{cases} \boxed{x = -3} \\ \boxed{x = 5} \end{cases} \end{cases}$$

$$\textcircled{\bullet} \quad \begin{array}{c|ccc|c} & 1 & -1 & -17 & -15 \\ -1 & & -1 & 2 & 15 \\ \hline & 1 & -2 & -15 & 0 \end{array}$$

$$\textcircled{2} \quad x^3 - 2x^2 - 9x + 18 = 0 \xRightarrow{\textcircled{\bullet}} (x-2) \cdot (x^2 - 9) \Rightarrow \begin{cases} x - 2 = 0 \Rightarrow \boxed{x = 2} \\ x^2 - 9 = 0 \Rightarrow \begin{cases} \boxed{x = -3} \\ \boxed{x = 3} \end{cases} \end{cases}$$

$$\textcircled{\bullet} \quad \begin{array}{c|ccc|c} & 1 & -2 & -9 & 18 \\ 2 & & 2 & 0 & -18 \\ \hline & 1 & 0 & -9 & 0 \end{array}$$

$$\textcircled{3} \quad x^4 - x^3 - 13x^2 + x + 12 = 0 \xRightarrow{\textcircled{\bullet}} (x-1) \cdot (x^3 - 13x - 12) \xRightarrow{\textcircled{*}} (x-1) \cdot (x+1) \cdot (x^2 - x - 12)$$

$$\Rightarrow \begin{cases} x - 1 = 0 \Rightarrow \boxed{x = 1} \\ x + 1 = 0 \Rightarrow \boxed{x = -1} \\ x^2 - x - 12 = 0 \Rightarrow \begin{cases} \boxed{x = -3} \\ \boxed{x = 4} \end{cases} \end{cases}$$

$$\textcircled{\bullet} \quad \begin{array}{c|cccc|c} & 1 & -1 & -13 & 1 & 12 \\ & & 1 & 0 & -13 & 12 \\ \hline & 1 & 0 & -13 & -12 & 0 \end{array}$$

$$\textcircled{*} \quad \begin{array}{c|ccc|c} & 1 & 0 & -13 & -12 \\ -1 & & -1 & 1 & 12 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

$$\textcircled{4} \quad x^5 + x^4 - 8x^3 + 14x^2 - 8x = 0 \Rightarrow x \cdot (x^4 + x^3 - 8x^2 + 14x - 8) = 0$$

$$\xRightarrow{\textcircled{\bullet}} x \cdot (x-1) \cdot (x^3 + 2x^2 - 6x + 8) \xRightarrow{\textcircled{*}} x \cdot (x-1) \cdot (x+4) \cdot (x^2 - 2x + 2)$$

$$\Rightarrow \begin{cases} \boxed{x = 0} \\ x - 1 = 0 \Rightarrow \boxed{x = 1} \\ x + 4 \Rightarrow \boxed{x = -4} \\ x^2 - 2x + 2 = 0 \Rightarrow \nexists \text{ Solución} \end{cases}$$

$$\textcircled{\bullet} \quad \begin{array}{c|cccc|c} & 1 & 1 & -8 & 14 & -8 \\ & & 1 & 2 & -6 & 8 \\ \hline & 1 & 2 & -6 & 8 & 0 \end{array}$$

$$\textcircled{*} \quad \begin{array}{c|ccc|c} & 1 & 2 & -6 & 8 \\ -4 & & -4 & 8 & -8 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$$\textcircled{5} \quad 4x^3 - 6x^2 - 6x + 4 = 0 \xRightarrow{\textcircled{\bullet}} (x+1) \cdot (4x^2 - 10x + 4) = 0$$

$$\Rightarrow \begin{cases} x + 1 = 0 \Rightarrow \boxed{x = -1} \\ 4x^2 - 10x + 4 = 0 \Rightarrow \begin{cases} \boxed{x = 1/2} \\ \boxed{x = 2} \end{cases} \end{cases}$$

$$\odot \quad \begin{array}{ccc|ccc} & & & 4 & -6 & -6 & 4 \\ & -1 & & & -4 & 10 & -4 \\ \hline & & & 4 & -10 & 4 & 0 \end{array}$$

$$\textcircled{6} \quad x^4 - 6x^3 + 12x^2 - 8x = 0 \implies x \cdot (x^3 - 6x^2 + 12x - 8) = 0 \xRightarrow{\odot} x \cdot (x-2) \cdot (x^2 - 4x + 4) = 0$$

$$\implies \begin{cases} x = 0 \\ x - 2 = 0 \implies x = 2 \\ x^2 - 4x + 4 = 0 \implies x = 2 \text{ d.} \end{cases}$$

$$\odot \quad \begin{array}{ccc|ccc} & & & 1 & -6 & 12 & -8 \\ & 2 & & & 2 & -8 & 8 \\ \hline & & & 1 & -4 & 4 & 0 \end{array}$$

$$\textcircled{7} \quad 6x^4 - 5x^3 - 14x^2 - x + 2 = 0 \xRightarrow{\odot} (x+1) \cdot (6x^3 - 11x^2 - 3x + 2) = 0$$

$$\xRightarrow{\otimes} (x+1) \cdot (x-2) \cdot (6x^2 + x - 1) \implies \begin{cases} x+1=0 \implies x=-1 \\ x-2=0 \implies x=2 \\ 6x^2 + x - 1 = 0 \implies \begin{cases} x = -1/2 \\ x = 1/3 \end{cases} \end{cases}$$

$$\odot \quad \begin{array}{cccc|ccc} & & & 6 & -5 & -14 & -1 & 2 \\ & -1 & & & -6 & 11 & 3 & -2 \\ \hline & & & 6 & -11 & -3 & 2 & 0 \end{array} \quad \otimes \quad \begin{array}{ccc|ccc} & & & 6 & -11 & -3 & 2 \\ & 2 & & & 12 & 2 & -2 \\ \hline & & & 6 & 1 & -1 & 0 \end{array}$$

$$\textcircled{8} \quad 4x^5 - 12x^4 - 3x^3 + 10x^2 - 3x \implies x \cdot (4x^4 - 12x^3 - 3x^2 + 10x - 3) = 0$$

$$\xRightarrow{\odot} x \cdot (x+1) \cdot (4x^3 - 16x^2 + 13x - 3) = 0 \xRightarrow{\otimes} x \cdot (x+1) \cdot (x-3) \cdot (4x^2 - 4x + 1) = 0$$

$$\implies \begin{cases} x = 0 \\ x+1=0 \implies x=-1 \\ x-3=0 \implies x=3 \\ 4x^2 - 4x + 1 = 0 \implies x = 1/2 \text{ d.} \end{cases}$$

$$\odot \quad \begin{array}{cccc|ccc} & & & 4 & -12 & -3 & 10 & -3 \\ & -1 & & & -4 & 16 & -13 & 3 \\ \hline & & & 4 & -16 & 13 & -3 & 0 \end{array} \quad \otimes \quad \begin{array}{ccc|ccc} & & & 4 & -16 & 13 & -3 \\ & 3 & & & 12 & -12 & 3 \\ \hline & & & 4 & -4 & 1 & 0 \end{array}$$

$$\textcircled{9} \quad x^3 - 4x = 0 \implies x \cdot (x^2 - 4) = 0 \implies \begin{cases} x = 0 \\ x^2 - 4 = 0 \implies x^2 = 4 \implies x = \pm 2 \end{cases}$$

$$\textcircled{10} \quad x^3 - x^2 - 5x - 3 = 0 \xRightarrow{\odot} (x+1) \cdot (x^2 - 2x - 3) = 0 \implies \begin{cases} x+1=0 \implies x=-1 \\ x^2 - 2x - 3 = 0 \implies \begin{cases} x = -1 \\ x = 3 \end{cases} \end{cases}$$

2.3. Ecuaciones Polinómicas de grado superior

$$\odot \begin{array}{ccc|ccc} & & & 1 & -1 & -5 & -3 \\ & & & & -1 & 2 & 3 \\ -1 & & & 1 & -2 & -3 & 0 \end{array}$$

$$(11) \quad x^3 - 2x^2 - 5x + 6 = 0 \xRightarrow{\odot} (x-1) \cdot (x^2 - x - 6) = 0 \Rightarrow \begin{cases} x - 1 = 0 \Rightarrow \boxed{x = 1} \\ x^2 - x - 6 = 0 \Rightarrow \begin{cases} \boxed{x = -2} \\ \boxed{x = 3} \end{cases} \end{cases}$$

$$\odot \begin{array}{ccc|ccc} & & & 1 & -2 & -5 & 6 \\ & & & & 1 & -1 & -6 \\ 1 & & & 1 & -1 & -6 & 0 \end{array}$$

$$(12) \quad x^4 - 9x^2 + 4x + 12 = 0 \xRightarrow{\odot} (x+1) \cdot (x^3 - x^2 - 8x + 12) = 0$$

$$\xRightarrow{(*)} (x+1) \cdot (x-2) \cdot (x^2 + x - 6) = 0 \Rightarrow \begin{cases} x + 1 = 0 \Rightarrow \boxed{x = -1} \\ x - 2 = 0 \Rightarrow \boxed{x = 2} \\ x^2 + x - 6 = 0 \Rightarrow \begin{cases} \boxed{x = -3} \\ \boxed{x = 2} \end{cases} \end{cases}$$

$$\odot \begin{array}{cccc|ccc} & & & 1 & 0 & -9 & 4 & 12 \\ & & & & -1 & 1 & 8 & -12 \\ -1 & & & 1 & -1 & -8 & 12 & 0 \end{array}$$

$$* \begin{array}{ccc|ccc} & & & 1 & -1 & -8 & 12 \\ & & & & 2 & 2 & -12 \\ 2 & & & 1 & 1 & -6 & 0 \end{array}$$

$$(13) \quad x^4 - 8x^3 + 14x^2 + 8x - 15 = 0 \xRightarrow{\odot} (x-1) \cdot (x^3 - 7x^2 + 7x + 15) = 0$$

$$\xRightarrow{(*)} (x-1) \cdot (x+1) \cdot (x^2 - 8x + 15) = 0 \Rightarrow \begin{cases} x - 1 = 0 \Rightarrow \boxed{x = 1} \\ x + 1 = 0 \Rightarrow \boxed{x = -1} \\ x^2 - 8x + 15 = 0 \Rightarrow \begin{cases} \boxed{x = 3} \\ \boxed{x = 5} \end{cases} \end{cases}$$

$$\odot \begin{array}{cccc|ccc} & & & 1 & -8 & 14 & 8 & -15 \\ & & & & 1 & -7 & 7 & 15 \\ 1 & & & 1 & -7 & 7 & 15 & 0 \end{array}$$

$$* \begin{array}{ccc|ccc} & & & 1 & -7 & 7 & 15 \\ & & & & -1 & 8 & -15 \\ -1 & & & 1 & -8 & 15 & 0 \end{array}$$

$$(14) \quad x^4 - 1 \Rightarrow x^4 = 1 \Rightarrow \boxed{x = \pm 1}$$

$$(15) \quad x^4 - x^3 - x^2 + x = 0 \Rightarrow x \cdot (x^3 - x^2 - x + 1) \xRightarrow{\odot} x \cdot (x-1) \cdot (x^2 - 1)$$

$$\Rightarrow \begin{cases} \boxed{x = 0} \\ x - 1 = 0 \Rightarrow \boxed{x = 1} \\ x^2 - 1 = 0 \Rightarrow \boxed{x = \pm 1} \end{cases}$$

$$\odot \begin{array}{ccc|ccc} & & & 1 & -1 & -1 & 1 \\ & & & & 1 & 0 & -1 \\ 1 & & & 1 & 0 & -1 & 0 \end{array}$$

$$(16) \quad x^4 - 2x^3 - 13x^2 + 38x - 24 = 0 \xRightarrow{\odot} (x-1) \cdot (x-2) \cdot (x^2 + x - 12) = 0$$

$$\Rightarrow \begin{cases} x - 1 = 0 \Rightarrow \boxed{x = 1} \\ x - 2 = 0 \Rightarrow \boxed{x = 2} \\ x^2 + x - 12 = 0 \Rightarrow \begin{cases} \boxed{x = -4} \\ \boxed{x = 3} \end{cases} \end{cases}$$

$$\odot \quad \begin{array}{ccc|ccc} 1 & 1 & -2 & -13 & 38 & -24 \\ & & 1 & -1 & -14 & 24 \\ \hline & 1 & -1 & -14 & 24 & 0 \end{array}$$

$$\otimes \quad \begin{array}{ccc|c} 2 & 1 & -1 & -14 & 24 \\ & & 2 & 2 & -24 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

$$(17) \quad x^5 - 5x^4 + 7x^3 - 3x^2 \Rightarrow x^2 \cdot (x^3 - 5x^2 + 7x - 3) = 0 \xRightarrow{\odot} x^2 \cdot (x - 1) \cdot (x^2 - 4x + 3) = 0$$

$$\Rightarrow \begin{cases} \boxed{x = 0} \\ x - 1 = 0 \Rightarrow \boxed{x = 1} \\ x^2 - 4x + 3 = 0 \Rightarrow \begin{cases} \boxed{x = 1} \\ \boxed{x = 3} \end{cases} \end{cases}$$

$$\odot \quad \begin{array}{ccc|c} 1 & 1 & -5 & 7 & -3 \\ & & 1 & -4 & 3 \\ \hline & 1 & -4 & 3 & 0 \end{array}$$

$$(18) \quad 3x^4 + 6x^3 + 6x^2 + 6x + 3 = 0 \xRightarrow{\odot} (x+1) \cdot (3x^3 + 3x^2 + 3x + 3) = 0 \xRightarrow{\otimes} (x+1)^2 \cdot (3x^2 + 3) = 0$$

$$\Rightarrow \begin{cases} (x+1)^2 = 0 \Rightarrow \boxed{x = -1} \\ 3x^2 + 3 = 0 \Rightarrow \nexists \text{ Solución} \end{cases}$$

$$\odot \quad \begin{array}{ccc|ccc} -1 & 3 & 6 & 6 & 6 & 3 \\ & & -3 & -3 & -3 & -3 \\ \hline & 3 & 3 & 3 & 3 & 0 \end{array}$$

$$\otimes \quad \begin{array}{ccc|c} -1 & 3 & 3 & 3 & 3 \\ & & -3 & 0 & -3 \\ \hline & 3 & 0 & 3 & 0 \end{array}$$

$$(19) \quad x^4 + 3x^3 + 4x^2 + 6x + 4 \xRightarrow{\odot} (x+1) \cdot (x^3 + 2x^2 + 2x + 4) = 0 \xRightarrow{\otimes} (x+1) \cdot (x+2) \cdot (x^2 + 2) = 0$$

$$\Rightarrow \begin{cases} x + 1 = 0 \Rightarrow \boxed{x = -1} \\ x + 2 = 0 \Rightarrow \boxed{x = -2} \\ x^2 + 2 = 0 \Rightarrow \nexists \text{ Solución} \end{cases}$$

$$\odot \quad \begin{array}{ccc|ccc} -1 & 1 & 3 & 4 & 6 & 4 \\ & & -1 & -2 & -2 & -4 \\ \hline & 1 & 2 & 2 & 4 & 0 \end{array}$$

$$\otimes \quad \begin{array}{ccc|c} -2 & 1 & 2 & 2 & 4 \\ & & -2 & 0 & -4 \\ \hline & 1 & 0 & 2 & 0 \end{array}$$

Ecuaciones con fracciones

3.1. Ecuaciones racionales

$$\textcircled{1} \quad -\frac{x^2}{3} - \frac{3x-1}{3} = 1$$

$$\textcircled{2} \quad \frac{(x-3)^2}{2} - x + x^2 = x - (x-2)$$

$$\textcircled{3} \quad \frac{1}{x-1} + 3x + 3x^2 - 2 = \frac{3}{x-1} + 3x^2$$

$$\textcircled{4} \quad (x-3)^2 - \frac{x-1}{3} = 2x$$

$$\textcircled{5} \quad \frac{x-3}{3} - \frac{1}{x-1} = 3x$$

$$\textcircled{6} \quad x - \frac{2}{x} + \frac{1}{2x} = 5x + 5$$

$$\textcircled{7} \quad \frac{x-3}{x} + 3x - \frac{5}{x} = 2x - \frac{3}{x} - 3$$

$$\textcircled{8} \quad 3x - \frac{8}{x} + (x-1) = 3 \cdot (x-2) - (x-5)$$

$$\textcircled{9} \quad (x-3) \cdot (x-2) + \frac{x \cdot (x-3)}{2} = (x-2)^2$$

$$\textcircled{10} \quad \frac{x^2-32}{4} = \frac{-28}{x^2-9}$$

$$\textcircled{11} \quad \frac{3}{x+3} + \frac{1}{6} = \frac{2}{x-2}$$

$$\textcircled{12} \quad \frac{x-1}{x+1} - \frac{3+x}{x} = 2$$

$$\textcircled{13} \quad \frac{x-1}{x+1} - \frac{3+x}{x-1} = 2$$

$$\textcircled{14} \quad x + \frac{1}{x-2} = 4$$

$$\textcircled{15} \quad x^2 - x = \frac{2}{9} - \frac{2x}{3}$$

$$\textcircled{16} \quad \frac{x^2}{3} + 2 = \frac{5x}{3}$$

$$\textcircled{17} \quad x + \frac{2}{x} = 3$$

$$\textcircled{18} \quad x - 2 = \frac{4x-8}{x}$$

$$\textcircled{19} \quad \frac{x}{2} + \frac{3}{x} = \frac{2x+9}{x}$$

$$\textcircled{20} \quad 2x - 2 = \frac{6x}{x-1} - 5$$

$$\textcircled{21} \quad \frac{x^2-5x+4}{8-x} = 5$$

$$\textcircled{22} \quad \frac{x-3}{x+5} = \frac{x-5}{x+3}$$

$$\textcircled{23} \quad \frac{x}{x+1} + \frac{x}{x-2} = 1$$

$$\textcircled{24} \quad \frac{2x}{x-2} = 1 + \frac{x+2}{2}$$

$$\textcircled{25} \quad \frac{3x}{8+4x} = \frac{x-2}{x}$$

$$\textcircled{26} \quad \frac{16x^3-12}{2x^2-4} = 6+8x$$

$$\textcircled{27} \quad \frac{2x^2-4}{x^2-4} = \frac{2x+1}{x+1}$$

$$\textcircled{28} \quad \frac{x+1}{x+2} + \frac{x-1}{x-2} = \frac{2x+1}{x+1}$$

$$\textcircled{29} \quad \frac{x+5}{x-5} + \frac{x-5}{x+5} = \frac{10}{3}$$

3.1. Ecuaciones racionales

$$(30) \quad \frac{3x^2 + 2}{x^2 + 2} = 4x^2 + 3$$

$$(31) \quad x \cdot (x + 1) - \left(x + \frac{x}{2}\right) = 0$$

$$(32) \quad 3x + 1 - \frac{3}{x} = \frac{1 + 3x}{4}$$

$$(33) \quad 2 + \frac{x + 4}{3} = \frac{4x + 4}{3} + \frac{2 - x}{x - 3}$$

$$(34) \quad x + \frac{1}{x} = \frac{6}{3x}$$

$$(35) \quad x - 2 = \frac{2x - 3}{x}$$

$$(36) \quad \frac{x}{3} + \frac{2}{x} = \frac{3x + 10}{3x}$$

$$(37) \quad x + 3 = \frac{2x + 1}{x - 1}$$

$$(38) \quad \frac{3}{x + \frac{1}{2 + \frac{x + 1}{x - 2}}} = \frac{1}{x}$$

$$(39) \quad \frac{\frac{x - 3}{2} - \frac{x - 3}{4}}{x - \frac{1}{1 - \frac{x - 1}{x + 1}}} = -\frac{1}{x}$$

$$(40) \quad \frac{1}{x + 1} + \frac{2}{x + 2} = \frac{9}{2}$$

$$(41) \quad \frac{x^2 - 5x + 4}{8 - x} = -\frac{1}{3}$$

$$(42) \quad \frac{x - 3}{x + 5} = \frac{x - 5}{x + 3}$$

$$(58) \quad (x - 2) \cdot x - \frac{x + 2}{3} - \frac{(x - 2) \cdot (x + 2)}{2} = (x - 2)^2 - 4$$

$$(59) \quad (x - 3)^2 - \frac{x - 2}{3} + (3 - x) \cdot (x - 1) = (x - 2)^2$$

$$(60) \quad \frac{2x}{x^2 + x - 2} + \frac{x^2 + 5}{x^2 - 3x + 2} = \frac{x - 2}{x^2 - 4} - \frac{4x}{x^2 - 2x + 1}$$

$$(43) \quad \frac{x}{x + 1} + \frac{x}{x - 2} = 1$$

$$(44) \quad \frac{2x}{x - 2} = 1 + \frac{x + 2}{2}$$

$$(45) \quad \frac{3x}{8 + 4x} = \frac{x - 2}{x}$$

$$(46) \quad \frac{16x^3 - 12}{2x^2 - 4} = 6 + 8x$$

$$(47) \quad \frac{2x^2 - 4}{x^2 - 4} = \frac{2x + 1}{x + 1}$$

$$(48) \quad \frac{x + 1}{x + 2} + \frac{x - 1}{x - 2} = \frac{2x + 1}{x + 1}$$

$$(49) \quad \frac{x^2}{x^2 + 2x + 1} = \frac{x + 2}{x + 1} - 2$$

$$(50) \quad \frac{x}{x - 1} + \frac{3}{x + 1} = \frac{x + 1}{x^2 - 1}$$

$$(51) \quad \frac{1 + x}{x - 3} - \frac{x}{x + 2} = \frac{3x + 5x^2}{x^2 - x - 6}$$

$$(52) \quad x - 1 = \frac{-4x}{x + 1} + 5$$

$$(53) \quad \frac{x + 2}{x + 1} + \frac{x + 1}{x + 2} - \frac{x + 5}{x + 2} = 0$$

$$(54) \quad \frac{x + 3}{x^2 - 2x + 1} - \frac{2}{x - 1} - \frac{2}{x + 1} = 0$$

$$(55) \quad -\frac{x^2}{x + 1} + \frac{2}{x - 1} - \frac{2}{x^2 - 1} = 0$$

$$(56) \quad x - 1 = \frac{-4x}{x + 1} + 5$$

$$(57) \quad \frac{x + 1}{x - 2} + \frac{x}{x + 2} = \frac{7x + 2}{x^2 - 4}$$

$$\textcircled{1} -\frac{x^2}{3} - \frac{3x-1}{3} = 1 \implies \frac{-x^2 - (3x-1)}{3} = \frac{3}{3} \implies -x^2 - 3x + 1 = 3$$

$$\implies x^2 + 3x + 2 = 0 \implies \begin{cases} x = -2 \\ x = -1 \end{cases}$$

$$\textcircled{2} \frac{(x-3)^2}{2} - x + x^2 = x - (x-2) \implies \frac{(x-3)^2}{2} - x + x^2 = x - x + 2 \implies \frac{(x-3)^2}{2} = 2 + x - x^2 \implies \frac{(x-3)^2}{2} = \frac{2 \cdot (2 + x - x^2)}{2} \implies (x-3)^2 = 4 + 2x - 2x^2$$

$$\implies x^2 - 6x + 9 = 4 + 2x - 2x^2 \implies 3x^2 - 8x + 5 = 0 \implies \begin{cases} x = 1 \\ x = 5/3 \end{cases}$$

$$\textcircled{3} \frac{1}{x-1} + 3x + 3x^2 - 2 = \frac{3}{x-1} + 3x^2 \implies \frac{1}{x-1} + 3x + 3x^2 - 2 = \frac{3}{x-1} + 3x^2$$

$$\implies \frac{1}{x-1} + 3x - 2 = \frac{3}{x-1} \implies \frac{1 + (x-1) \cdot (3x-2)}{x-1} = \frac{3}{x-1}$$

$$\implies 1 + 3x^2 - 2x - 3x + 2 = 3 \implies 3x^2 - 5x = 0 \implies x \cdot (3x - 5) = 0$$

$$\implies \begin{cases} x = 0 \\ 3x - 5 = 0 \implies x = 5/3 \end{cases}$$

$$\textcircled{4} (x-3)^2 - \frac{x-1}{3} = 2x \implies x^2 - 6x + 9 - \frac{x-1}{3} = 2x \implies x^2 - 8x + 9 = \frac{x-1}{3}$$

$$\implies \frac{3 \cdot (x^2 - 8x + 9)}{3} = \frac{x-1}{3} \implies 3x^2 - 24x + 27 = x - 1 \implies 3x^2 - 25x + 28 = 0$$

$$\implies \begin{cases} x = 7 \\ x = \frac{4}{3} \end{cases}$$

$$\textcircled{5} \frac{x-3}{3} - \frac{1}{x-1} = 3x \implies \frac{(x-3) \cdot (x-1) - 3}{3 \cdot (x-1)} = \frac{3x \cdot 3 \cdot (x-1)}{3 \cdot (x-1)}$$

$$\implies x^2 - x - 3x + 3 - 3 = 9x^2 - 9x \implies 8x^2 - 5x = 0 \implies x \cdot (8x - 5) = 0$$

$$\implies \begin{cases} x = 0 \\ 8x - 5 = 0 \implies x = 5/8 \end{cases}$$

$$\textcircled{6} x - \frac{2}{x} + \frac{1}{2x} = 5x + 5 \implies -\frac{2}{x} + \frac{1}{2x} = 4x + 5 \implies \frac{-4+1}{2x} = \frac{2x \cdot (4x+5)}{2x}$$

$$\implies -4+1 = 8x^2 + 10x \implies 8x^2 + 10x + 3 = 0 \implies \begin{cases} x = -1/2 \\ x = -3/4 \end{cases}$$

$$\textcircled{7} \frac{x-3}{x} + 3x - \frac{5}{x} = 2x - \frac{3}{x} - 3 \implies \frac{x-3}{x} - \frac{5}{x} = -x - \frac{3}{x} - 3$$

$$\implies \frac{x-3-5}{x} = \frac{-x^2-3-3x}{x} \implies x-3-5 = -x^2-3-3x \implies x^2+4x-5 = 0$$

3.1. Ecuaciones racionales

$$\Rightarrow \begin{cases} x = -5 \\ x = 1 \end{cases}$$

$$\textcircled{8} \quad 3x - \frac{8}{x} + (x-1) = 3 \cdot (x-2) - (x-5) \Rightarrow \cancel{3x} - \frac{8}{x} + x - 1 = \cancel{3x} - 6 - x + 5$$

$$\Rightarrow 2x = \frac{8}{x} \Rightarrow 2x^2 = 8 \Rightarrow x^2 = 4 \Rightarrow \begin{cases} x = -2 \\ x = 2 \end{cases}$$

$$\textcircled{9} \quad (x-3) \cdot (x-2) + \frac{x \cdot (x-3)}{2} = (x-2)^2$$

$$\Rightarrow \frac{2 \cdot (x-3) \cdot (x-2) + x \cdot (x-3)}{2} = \frac{2 \cdot (x-2)^2}{2}$$

$$\Rightarrow 2x^2 - 4x - 6x + 12 + x^2 - 3x = 2x^2 - 8x + 8 \Rightarrow x^2 - 5x + 4 = 0$$

$$\Rightarrow \begin{cases} x = 1 \\ x = 4 \end{cases} \Rightarrow$$

$$\textcircled{10} \quad \frac{x^2 - 32}{4} = \frac{-28}{x^2 - 9} \Rightarrow (x^2 - 32) \cdot (x^2 - 9) = -112 \Rightarrow x^4 - 9x^2 - 32x^2 + 288 = -112$$

$$\Rightarrow x^4 - 41x^2 + 400 = 0 \xrightarrow[x^2=t]{x^4=t^2} t^2 - 41t + 400 = 0 \Rightarrow \begin{cases} t = 16 = x^2 \Rightarrow x = \pm 4 \\ t = 25 = x^2 \Rightarrow x = \pm 5 \end{cases}$$

$$\textcircled{11} \quad \frac{3}{x+3} + \frac{1}{6} = \frac{2}{x-2} \Rightarrow \frac{18 \cdot (x-2) + (x+3) \cdot (x-2)}{6 \cdot (x+3) \cdot (x-2)} = \frac{12 \cdot (x+3)}{6 \cdot (x+3) \cdot (x-2)}$$

$$\Rightarrow 18x - 36 + x^2 - 2x + 3x - 6 = 12x + 36 \Rightarrow x^2 + 7x - 78 = 0$$

$$\Rightarrow \begin{cases} x = -13 \\ x = 6 \end{cases}$$

$$\textcircled{12} \quad \frac{x-1}{x+1} - \frac{3+x}{x} = 2 \Rightarrow \frac{x \cdot (x-1) - (3+x) \cdot (x+1)}{x \cdot (x+1)} = \frac{2x \cdot (x+1)}{x \cdot (x+1)}$$

$$\Rightarrow x^2 - x - (3x + 3 + x^2 + x) = 2x^2 + 2x \Rightarrow x^2 - x - 3x - 3 - x^2 - x = 2x^2 + 2x$$

$$\Rightarrow 2x^2 + 7x + 3 = 0 \Rightarrow \begin{cases} x = -3 \\ x = -1/2 \end{cases}$$

$$\textcircled{13} \quad \frac{x-1}{x+1} - \frac{3+x}{x-1} = 2 \Rightarrow \frac{(x-1)^2 - (3+x) \cdot (x+1)}{(x+1) \cdot (x-1)} = \frac{2 \cdot (x+1) \cdot (x-1)}{(x+1) \cdot (x-1)}$$

$$\Rightarrow x^2 - 2x + 1 - (3x + 3 + x^2 + x) = 2 \cdot (x^2 - 1) \Rightarrow x^2 - 2x + 1 - 3x - 3 - x^2 - x = 2x^2 - 2$$

$$\Rightarrow 2x^2 + 6x = 0 \Rightarrow 2x \cdot (x+3) = 0 \Rightarrow \begin{cases} 2x = 0 \Rightarrow x = 0 \\ x + 3 = 0 \Rightarrow x = -3 \end{cases}$$

$$\textcircled{14} \quad x + \frac{1}{x-2} = 4 \Rightarrow \frac{x \cdot (x-2) + 1}{x-2} = \frac{4 \cdot (x-2)}{x-2} \Rightarrow x^2 - 2x + 1 = 4x - 8$$

$$\Rightarrow x^2 - 6x + 9 \Rightarrow x = 3$$

$$(15) \quad x^2 - x = \frac{2}{9} - \frac{2x}{3} \Rightarrow \frac{9x^2 - 9x}{9} = \frac{2 - 6x}{9} \Rightarrow 9x^2 - 9x = 2 - 6x \Rightarrow 9x^2 - 3x - 2 = 0$$

$$\Rightarrow \begin{cases} x = -1/3 \\ x = 2/3 \end{cases}$$

$$(16) \quad \frac{x^2}{3} + 2 = \frac{5x}{3} \Rightarrow \frac{x^2 + 6}{3} = \frac{5x}{3} \Rightarrow x^2 - 5x + 6 = 0 \Rightarrow \begin{cases} x = 2 \\ x = 3 \end{cases}$$

$$(17) \quad x + \frac{2}{x} = 3 \Rightarrow \frac{x^2 + 2}{x} = \frac{3x}{x} \Rightarrow x^2 - 3x + 2 = 0 \Rightarrow \begin{cases} x = 1 \\ x = 2 \end{cases}$$

$$(18) \quad x - 2 = \frac{4x - 8}{x} \Rightarrow \frac{x^2 - 2x}{x} = \frac{4x - 8}{x} \Rightarrow x^2 - 6x + 8 = 0 \Rightarrow \begin{cases} x = 2 \\ x = 4 \end{cases}$$

$$(19) \quad \frac{x}{2} + \frac{3}{x} = \frac{2x + 9}{x} \Rightarrow \frac{x^2 + 6}{2x} = \frac{2 \cdot (2x + 9)}{2x} \Rightarrow x^2 + 6 = 4x + 18$$

$$\Rightarrow x^2 - 4x - 12 = 0 \Rightarrow \begin{cases} x = -2 \\ x = 6 \end{cases}$$

$$(20) \quad 2x - 2 = \frac{6x}{x - 1} - 5 \Rightarrow \frac{(2x - 2) \cdot (x - 1)}{x - 1} = \frac{6x - 5 \cdot (x - 1)}{x - 1}$$

$$\Rightarrow 2x^2 - 2x - 2x + 2 = 6x - 5x + 5 \Rightarrow 2x^2 - 5x - 3 = 0 \Rightarrow \begin{cases} x = -1/2 \\ x = 3 \end{cases}$$

$$(21) \quad \frac{x^2 - 5x + 4}{8 - x} = 5 \Rightarrow x^2 - 5x + 4 = 5 \cdot (8 - x) \Rightarrow x^2 - 5x + 4 = 40 - 5x$$

$$\Rightarrow x^2 - 36 = 0 \Rightarrow x = \pm 6$$

$$(22) \quad \frac{x - 3}{x + 5} = \frac{x - 5}{x + 3} \Rightarrow (x - 3) \cdot (x + 3) = (x - 5) \cdot (x + 5) \Rightarrow x^2 - 9 = x^2 - 25$$

$$\Rightarrow 16 = 0 \Rightarrow \nexists \text{ Solución}$$

$$(23) \quad \frac{x}{x + 1} + \frac{x}{x - 2} = 1 \Rightarrow \frac{x \cdot (x - 2) + x \cdot (x + 1)}{(x + 1) \cdot (x - 2)} = \frac{(x + 1) \cdot (x - 2)}{(x + 1) \cdot (x - 2)}$$

$$\Rightarrow x^2 - 2x + x^2 + x = x^2 + x - 2x - 2 \Rightarrow x^2 + 1 = 0 \Rightarrow x = \pm \sqrt{-1} \Rightarrow \nexists \text{ Solución}$$

$$(24) \quad \frac{2x}{x - 2} = 1 + \frac{x + 2}{2} \Rightarrow \frac{4x}{2 \cdot (x - 2)} = \frac{2 \cdot (x - 2) + (x + 2) \cdot (x - 2)}{2 \cdot (x - 2)}$$

$$\Rightarrow 4x = 2x - 4 + x^2 - 4 \Rightarrow x^2 - 2x - 8 = 0 \Rightarrow \begin{cases} x = -2 \\ x = 4 \end{cases}$$

$$(25) \quad \frac{3x}{8 + 4x} = \frac{x - 2}{x} \Rightarrow 3x^2 = (x - 2) \cdot (8 + 4x) \Rightarrow 3x^2 = 8x + 4x^2 - 16 - 8x$$

$$\Rightarrow x^2 - 16 = 0 \Rightarrow x = \pm 4$$

3.1. Ecuaciones racionales

$$\begin{aligned} 26 \quad \frac{16x^3 - 12}{2x^2 - 4} = 6 + 8x &\implies \frac{16x^3 - 12}{2x^2 - 4} = \frac{(6 + 8x) \cdot (2x^2 - 4)}{2x^2 - 4} \\ &\implies \cancel{16x^3} - 12 = 12x^2 - 24 + \cancel{16x^3} - 32x \implies 12x^2 - 32x - 12 = 0 \implies \\ &\quad \begin{cases} x = -1/3 \\ x = 3 \end{cases} \end{aligned}$$

$$\begin{aligned} 27 \quad \frac{2x^2 - 4}{x^2 - 4} = \frac{2x + 1}{x + 1} &\implies (2x^2 - 4) \cdot (x + 1) = (2x + 1) \cdot (x^2 - 4) \\ &\implies \cancel{2x^3} + 2x^2 - 4x - 4 = \cancel{2x^3} - 8x + x^2 - 4 \implies x^2 + 4x = 0 \\ &\implies x \cdot (x + 4) = 0 \implies \begin{cases} x = 0 \\ x + 4 = 0 \implies x = -4 \end{cases} \end{aligned}$$

$$\begin{aligned} 28 \quad \frac{x + 1}{x + 2} + \frac{x - 1}{x - 2} = \frac{2x + 1}{x + 1} &\implies \frac{(x + 1)^2 \cdot (x - 2) + (x - 1) \cdot (x + 1) \cdot (x + 2)}{(x + 1) \cdot (x + 2) \cdot (x - 2)} = \frac{(2x + 1) \cdot (x + 2) \cdot (x - 2)}{(x + 1) \cdot (x + 2) \cdot (x - 2)} \\ &\implies (x^2 + 2x + 1) \cdot (x - 2) + (x^2 - 1) \cdot (x + 2) = (2x + 1) \cdot (x^2 - 4) \\ &\implies \cancel{x^3} - 2x^2 + \cancel{2x^2} - 4x + x - 2 + \cancel{x^3} + 2x^2 - x - 2 = \cancel{2x^3} - 8x + x^2 - 4 \\ &\implies x^2 + 4x = 0 \implies x \cdot (x + 4) = 0 \implies \begin{cases} x = 0 \\ x + 4 = 0 \implies x = -4 \end{cases} \end{aligned}$$

$$\begin{aligned} 29 \quad \frac{x + 5}{x - 5} + \frac{x - 5}{x + 5} = \frac{10}{3} &\implies \frac{3 \cdot (x + 5)^2 + 3 \cdot (x - 5)^2}{3 \cdot (x - 5) \cdot (x + 5)} = \frac{10 \cdot (x - 5) \cdot (x + 5)}{3 \cdot (x - 5) \cdot (x + 5)} \\ &\implies 3x^2 + \cancel{30x} + 75 + 3x^2 - \cancel{30x} + 75 = 10x^2 - 250 \implies 4x^2 - 400 = 0 \\ &\implies x^2 = 100 \implies x = \pm 10 \end{aligned}$$

$$\begin{aligned} 30 \quad \frac{3x^2 + 2}{x^2 + 2} = 4x^2 + 3 &\implies 3x^2 + 2 = (x^2 + 2) \cdot (4x^2 + 3) \implies \cancel{3x^2} + 2 = 4x^4 + \cancel{3x^2} + 8x^2 + 6 \\ &\implies 4x^4 + 8x^2 + 4 = 0 \xrightarrow[x^4=t^2]{x^2=t} 4t^2 + 8x + 4 = 0 \implies \nexists \text{ Solución} \end{aligned}$$

$$\begin{aligned} 31 \quad x \cdot (x + 1) - \left(x + \frac{x}{2}\right) = 0 &\implies \frac{2x \cdot (x + 1) - (2x + x)}{2} = 0 \implies 2x^2 + 2x - 3x = 0 \\ &\implies 2x^2 - x = 0 \implies x \cdot (2x - 1) = 0 \implies \begin{cases} x = 0 \\ 2x - 1 = 0 \implies x = 1/2 \end{cases} \end{aligned}$$

$$\begin{aligned} 32 \quad 3x + 1 - \frac{3}{x} = \frac{1 + 3x}{4} &\implies \frac{4x \cdot (3x + 1) - 12}{4x} = \frac{x \cdot (1 + 3x)}{4x} \\ &\implies 12x^2 + 4x - 12 = x + 3x^2 \implies 9x^2 + 3x - 12 = 0 \implies \begin{cases} x = -4/3 \\ x = 1 \end{cases} \end{aligned}$$

$$33 \quad 2 + \frac{x + 4}{3} = \frac{4x + 4}{3} + \frac{2 - x}{x - 3}$$

$$\Rightarrow \frac{6 \cdot (x-3) + (x+4) \cdot (x-3)}{3 \cdot (x-3)} = \frac{(4x+4) \cdot (x-3) + 3 \cdot (2-x)}{3 \cdot (x-3)}$$

$$\Rightarrow 6x - 18 + x^2 - 3x + 4x - 12 = 4x^2 - 12x + 4x - 12 + 6 - 3x$$

$$\Rightarrow 3x^2 - 18x + 24 = 0 \Rightarrow \begin{cases} x = 2 \\ x = 4 \end{cases}$$

$$(34) \quad x + \frac{1}{x} = \frac{6}{3x} \Rightarrow \frac{3x^2 + 3}{3x} = \frac{6}{3x} \Rightarrow 3x^2 + 3 = 6 \Rightarrow x^2 = 1 \Rightarrow \boxed{x = \pm 1}$$

$$(35) \quad x - 2 = \frac{2x - 3}{x} \Rightarrow x \cdot (x - 2) = 2x - 3 \Rightarrow x^2 - 2x = 2x - 3 \Rightarrow x^2 - 4x + 3 = 0$$

$$\Rightarrow \begin{cases} x = 1 \\ x = 3 \end{cases}$$

$$(36) \quad \frac{x}{3} + \frac{2}{x} = \frac{3x + 10}{3x} \Rightarrow \frac{x^2 + 6}{3x} = \frac{3x + 10}{3x} \Rightarrow x^2 + 6 = 3x + 10 \Rightarrow x^2 - 3x - 4 = 0$$

$$\Rightarrow \begin{cases} x = -1 \\ x = 4 \end{cases}$$

$$(37) \quad x + 3 = \frac{2x + 1}{x - 1} \Rightarrow (x + 3) \cdot (x - 1) = 2x + 1 \Rightarrow x^2 - x + 3x - 3 = 2x + 1$$

$$\Rightarrow x^2 - 4 = 0 \Rightarrow x^2 = 4 \Rightarrow \boxed{x = \pm 2}$$

$$(38) \quad \frac{3}{x + \frac{1}{2 + \frac{x+1}{x-2}}} = \frac{1}{x} \Rightarrow \frac{3}{x + \frac{1}{\frac{2 \cdot (x-2) + x+1}{x-2}}} = \frac{1}{x} \Rightarrow \frac{3}{x + \frac{1}{\frac{3x-3}{x-2}}} = \frac{1}{x}$$

$$\Rightarrow \frac{3}{x + \frac{x-2}{3x-3}} = \frac{1}{x} \Rightarrow \frac{3}{\frac{x \cdot (3x-3) + x-2}{3x-3}} = \frac{1}{x} \Rightarrow \frac{3}{\frac{3x^2 - 2x - 2}{3x-3}} = \frac{1}{x}$$

$$\Rightarrow \frac{3 \cdot (3x-3)}{3x^2 - 2x - 2} = \frac{1}{x} \Rightarrow 3x \cdot (3x-3) = 3x^2 - 2x - 2 \Rightarrow 9x^2 - 9x = 3x^2 - 2x - 2$$

$$\Rightarrow 6x^2 - 7x + 2 = 0 \Rightarrow \begin{cases} x = 1/2 \\ x = 2/3 \end{cases}$$

$$(39) \quad \frac{\frac{x-3}{2} - \frac{x-3}{4}}{x - \frac{1}{1 - \frac{x-1}{x+1}}} = -\frac{1}{x} \Rightarrow \frac{\frac{2 \cdot (x-3) - (x-3)}{4}}{x - \frac{1}{\frac{x+1 - (x-1)}{x+1}}} = -\frac{1}{x} \Rightarrow \frac{\frac{x-3}{4}}{x - \frac{1}{\frac{2}{x+1}}} = -\frac{1}{x}$$

$$\Rightarrow \frac{\frac{x-3}{4}}{x - \frac{x+1}{2}} = -\frac{1}{x} \Rightarrow \frac{\frac{x-3}{4}}{\frac{2x - (x+1)}{2}} = -\frac{1}{x} \Rightarrow \frac{\frac{x-3}{4}}{\frac{x-1}{2}} = -\frac{1}{x}$$

$$\Rightarrow \frac{2 \cdot (x-3)}{4 \cdot (x-1)} = -\frac{1}{x} \Rightarrow 2x \cdot (x-3) = -4 \cdot (x-1) \Rightarrow 2x^2 - 6x = -4x + 4$$

3.1. Ecuaciones racionales

$$\Rightarrow 2x^2 - 2x - 4 = 0 \Rightarrow \begin{cases} x = -1 \\ x = 2 \end{cases}$$

$$\textcircled{40} \frac{1}{x+1} + \frac{2}{x+2} = \frac{9}{2} \Rightarrow \frac{2 \cdot (x+2) + 4 \cdot (x+1)}{2} = \frac{9 \cdot (x+1) \cdot (x+2)}{2}$$

$$\Rightarrow 2x + 4 + 4x + 4 = 9 \cdot (x^2 + 2x + x + 2) \Rightarrow 6x + 8 = 9x^2 + 27x + 18$$

$$\Rightarrow 9x^2 + 21x + 10 = 0 \Rightarrow \begin{cases} x = -5/3 \\ x = -2/3 \end{cases}$$

$$\textcircled{41} \frac{x^2 - 5x + 4}{8 - x} = -\frac{1}{3} \Rightarrow 3 \cdot (x^2 - 5x + 4) = -(8 - x) \Rightarrow 3x^2 - 15x + 12 = -8 + x$$

$$\Rightarrow 3x^2 - 16x + 20 = 0 \Rightarrow \begin{cases} x = 2 \\ x = 10/3 \end{cases}$$

$$\textcircled{42} \frac{x-3}{x+5} = \frac{x-5}{x+3} \Rightarrow (x-3) \cdot (x+3) = (x-5) \cdot (x+5) \Rightarrow x^2 - 9 = x^2 - 25$$

$$\Rightarrow 16 = 0 \Rightarrow \nexists \text{ Solución}$$

$$\textcircled{43} \frac{x}{x+1} + \frac{x}{x-2} = 1 \Rightarrow \frac{x \cdot (x-2) + x \cdot (x+1)}{(x+1) \cdot (x-2)} = \frac{(x+1) \cdot (x-2)}{(x+1) \cdot (x-2)}$$

$$\Rightarrow x^2 - 2x + x^2 + x = x^2 - 2x + x - 2 \Rightarrow x^2 + 2 = 0 \Rightarrow \nexists \text{ Solución}$$

$$\textcircled{44} \frac{2x}{x-2} = 1 + \frac{x+2}{2} \Rightarrow \frac{4x}{2 \cdot (x-2)} = \frac{2 \cdot (x-2) + (x+2) \cdot (x-2)}{2 \cdot (x-2)}$$

$$\Rightarrow 4x = 2x - 4 + x^2 - 4 \Rightarrow x^2 - 2x - 8 = 0 \Rightarrow \begin{cases} x = -2 \\ x = 4 \end{cases}$$

$$\textcircled{45} \frac{3x}{8+4x} = \frac{x-2}{x} \Rightarrow 3x^2 = (x-2) \cdot (8+4x) \Rightarrow 3x^2 = 8x + 4x^2 - 16 - 8x$$

$$\Rightarrow x^2 - 16 = 0 \Rightarrow x = \pm 4$$

$$\textcircled{46} \frac{16x^3 - 12}{2x^2 - 4} = 6 + 8x \Rightarrow 16x^3 - 12 = (6 + 8x) \cdot (2x^2 - 4)$$

$$\Rightarrow 16x^3 - 12 = 12x^2 - 24 + 16x^3 - 32x \Rightarrow 12x^2 - 32x - 12 = 0$$

$$\Rightarrow \begin{cases} x = -1/3 \\ x = 3 \end{cases}$$

$$\textcircled{47} \frac{2x^2 - 4}{x^2 - 4} = \frac{2x + 1}{x + 1} \Rightarrow (2x^2 - 4) \cdot (x + 1) = (2x + 1) \cdot (x^2 - 4)$$

$$\Rightarrow 2x^3 + 2x^2 - 4x - 4 = 2x^3 - 8x + x^2 - 4 \Rightarrow x^2 + 4x = 0$$

$$\Rightarrow x \cdot (x + 4) = 0 \Rightarrow \begin{cases} x = 0 \\ x + 4 = 0 \Rightarrow x = -4 \end{cases}$$

$$\begin{aligned}
48 \quad & \frac{x+1}{x+2} + \frac{x-1}{x-2} = \frac{2x+1}{x+1} \\
\Rightarrow & \frac{(x+1)^2 \cdot (x-2) + (x-1) \cdot (x+1) \cdot (x+2)}{(x+2) \cdot (x-2) \cdot (x+1)} = \frac{(2x+1) \cdot (x+2) \cdot (x-2)}{(x+2) \cdot (x-2) \cdot (x+1)} \\
\Rightarrow & (x^2 + 2x + 1) \cdot (x-2) + (x^2 - 1) \cdot (x+2) = (2x+1) \cdot (x^2 - 4) \\
\Rightarrow & \cancel{x^3} - 2\cancel{x^2} + 2\cancel{x^2} - 4x + x - 2 + \cancel{x^3} + 2x^2 - x - 2 = 2\cancel{x^3} - 8x + x^2 - 4 \\
\Rightarrow & x^2 + 4x = 0 \Rightarrow x \cdot (x+4) = 0 \Rightarrow \begin{cases} x = 0 \\ x + 4 = 0 \Rightarrow x = -4 \end{cases}
\end{aligned}$$

$$\begin{aligned}
49 \quad & \frac{x^2}{x^2 + 2x + 1} = \frac{x+2}{x+1} - 2 \Rightarrow \frac{x^2}{(x+1)^2} = \frac{x+2}{x+1} - 2 \\
\Rightarrow & \frac{x^2}{(x+1)^2} = \frac{(x+2) \cdot (x+1) - 2 \cdot (x+1)^2}{(x+1)^2} \\
\Rightarrow & x^2 = x^2 + x + 2x + 2 - 2x^2 - 4x - 2 \Rightarrow 2x^2 + x = 0 \Rightarrow x \cdot (2x+1) = 0 \\
\Rightarrow & \begin{cases} x = 0 \\ x = -1/2 \end{cases}
\end{aligned}$$

$$\begin{aligned}
50 \quad & \frac{x}{x-1} + \frac{3}{x+1} = \frac{x+1}{x^2-1} \Rightarrow \frac{x}{x-1} + \frac{3}{x+1} = \frac{x+1}{(x+1) \cdot (x-1)} \\
\Rightarrow & \frac{x \cdot (x+1) + 3 \cdot (x-1)}{(x+1) \cdot (x-1)} = \frac{x+1}{(x+1) \cdot (x-1)} \Rightarrow x^2 + x + 3x - 3 = x + 1 \\
\Rightarrow & x^2 + 3x - 4 = 0 \Rightarrow \begin{cases} x = -4 \\ x = 1 \end{cases}
\end{aligned}$$

$$\begin{aligned}
51 \quad & \frac{1+x}{x-3} - \frac{x}{x+2} = \frac{3x+5x^2}{x^2-x-6} \Rightarrow \frac{1+x}{x-3} - \frac{x}{x+2} = \frac{3x+5x^2}{(x-3) \cdot (x+2)} \\
\Rightarrow & \frac{(1+x) \cdot (x+2) - x \cdot (x-3)}{(x-3) \cdot (x+2)} = \frac{3x+5x^2}{(x-3) \cdot (x+2)} \\
\Rightarrow & x + 2 + x^2 + 2x - x^2 + 3x = 3x + 5x^2 \Rightarrow 5x^2 - 3x - 2 = 0 \\
\Rightarrow & \begin{cases} x = -2/5 \\ x = 1 \end{cases}
\end{aligned}$$

$$\begin{aligned}
52 \quad & x - 1 = \frac{-4x}{x+1} + 5 \Rightarrow \frac{(x-1) \cdot (x+1)}{x+1} = \frac{-4x + 5 \cdot (x+1)}{x+1} \\
\Rightarrow & x^2 - 1 = -4x + 5x + 5 \Rightarrow x^2 - x - 6 = 0 \Rightarrow \begin{cases} x = -2 \\ x = 3 \end{cases}
\end{aligned}$$

$$\begin{aligned}
53 \quad & \frac{x+2}{x+1} + \frac{x+1}{x+2} - \frac{x+5}{x+2} = 0 \Rightarrow \frac{(x+2)^2 + (x+1)^2 - (x+5) \cdot (x+1)}{(x+1) \cdot (x+2)} = 0 \\
\Rightarrow & x^2 + 4x + 4 + x^2 + 2x + 1 - (x^2 + x + 5x + 5) = 0 \\
\Rightarrow & x^2 + 4x + 4 + x^2 + 2x + 1 - x^2 - x - 5x - 5 = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0
\end{aligned}$$

3.1. Ecuaciones racionales

$$\begin{aligned}
 54 \quad & \frac{x+3}{x^2-2x+1} - \frac{2}{x-1} - \frac{2}{x+1} = 0 \implies \frac{x+3}{(x-1)^2} - \frac{2}{x-1} - \frac{2}{x+1} = 0 \\
 & \implies \frac{(x+3) \cdot (x+1) - 2 \cdot (x-1) \cdot (x+1) - 2 \cdot (x-1)^2}{(x+1) \cdot (x-1)^2} \\
 & \implies x^2 + x + 3x + 3 - (2x^2 - 2) - (2x^2 - 4x + 2) = 0 \\
 & \implies x^2 + x + 3x + 3 - 2x^2 + 2 - 2x^2 + 4x - 2 = 0 \implies -3x^2 + 6x - 3 = 0 \\
 & \implies x^2 - 2x + 1 = 0 \implies \begin{cases} x = -1/3 \\ x = 3 \end{cases} \implies x \neq -1 \implies \nexists \text{ Solución}
 \end{aligned}$$

$$\begin{aligned}
 55 \quad & -\frac{x^2}{x+1} + \frac{2}{x-1} - \frac{2}{x^2-1} = 0 \implies -\frac{x^2}{x+1} + \frac{2}{x-1} - \frac{2}{(x+1) \cdot (x-1)} = 0 \\
 & \implies \frac{-x^2 \cdot (x-1) + 2 \cdot (x+1) - 2}{x^2-1} = 0 \implies -x^3 + x^2 + 2x + 2 - 2 = 0 \\
 & \implies x^3 - x^2 - 2x = 0 \implies x \cdot (x^2 - x - 2) = 0 \implies \begin{cases} x = 0 \\ x \neq -1 \\ [1mm] x = 2 \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 56 \quad & x-1 = \frac{-4x}{x+1} + 5 \implies \frac{(x-1) \cdot (x+1)}{x+1} = \frac{-4x + 5 \cdot (x+1)}{x+1} \\
 & \implies x^2 - 1 = -4x + 5x + 5 \implies x^2 - x - 6 = 0 \implies \begin{cases} x = -2 \\ x = 3 \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 57 \quad & \frac{x+1}{x-2} + \frac{x}{x+2} = \frac{7x+2}{x^2-4} \implies \frac{x+1}{x-2} + \frac{x}{x+2} = \frac{7x+2}{(x+2) \cdot (x-2)} \\
 & \implies \frac{(x+1) \cdot (x+2) + x \cdot (x-2)}{(x+2) \cdot (x-2)} = \frac{7x+2}{(x+2) \cdot (x-2)} \\
 & \implies x^2 + 2x + x + 2 + x^2 - 2x = 7x + 2 \implies 2x^2 - 6x = 0 \implies 2x \cdot (x-3) = 0 \\
 & \implies \begin{cases} 2x = 0 \implies x = 0 \\ x - 3 = 0 \implies x = 3 \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 58 \quad & (x-2) \cdot x - \frac{x+2}{3} - \frac{(x-2) \cdot (x+2)}{2} = (x-2)^2 - 4 \\
 & \implies \frac{6x \cdot (x-2) - 2 \cdot (x+2) - 3 \cdot (x^2-4)}{6} = \frac{6 \cdot (x-2)^2 - 24}{6} \\
 & \implies 6x^2 - 12x - 2x - 4 - 3x^2 + 12 = 6(x^2 - 4x + 4) - 24 \\
 & \implies 6x^2 - 12x - 2x - 4 - 3x^2 + 12 = 6x^2 - 24x + 24 - 24 \\
 & \implies 3x^2 - 10x - 8 = 0 \implies \begin{cases} x = -2/3 \\ x = 4 \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 59 \quad & (x-3)^2 - \frac{x-2}{3} + (3-x) \cdot (x-1) = (x-2)^2 \\
 & \implies x^2 - 6x + 9 - \frac{x-2}{3} + 3x - 3 - x^2 + x = x^2 - 4x + 4
 \end{aligned}$$

$$\Rightarrow x^2 - 2x - 2 + \frac{x-2}{3} = 0 \Rightarrow \frac{3 \cdot (x^2 - 2x - 2) + x - 2}{3} = 0$$

$$\Rightarrow 3x^2 - 6x - 6 + x - 2 = 0 \Rightarrow 3x^2 - 5x - 8 = 0 \Rightarrow \begin{cases} x = -1 \\ x = 8/3 \end{cases}$$

60 $\frac{2x}{x^2+x-2} + \frac{x^2+5}{x^2-3x+2} = \frac{x-2}{x^2-4} - \frac{4x}{x^2-2x+1}$

$$\Rightarrow \frac{2x}{(x+2) \cdot (x-1)} + \frac{x^2+5}{(x-1) \cdot (x-2)} = \frac{x-2}{(x+2) \cdot (x-2)} - \frac{4x}{(x-1)^2}$$

$$\Rightarrow \frac{2x(x-1)(x-2) + (x^2+5)(x-1)(x+2)}{(x-1)^2 \cdot (x+2) \cdot (x-2)} = \frac{(x-2)(x-1)^2 - 4x(x+2)(x-2)}{(x-1)^2 \cdot (x+2) \cdot (x-2)}$$

$$\Rightarrow 2x \cdot (x^2 - 3x + 2) + (x^2 + 5) \cdot (x^2 + x - 2) = (x-2) \cdot (x^2 - 2x + 1) - 4x \cdot (x^2 - 4)$$

$$\Rightarrow 2x^3 - 6x^2 + 4x + x^4 + x^3 - 2x^2 + 5x^2 + 5x - 10 = x^3 - 2x^2 + x - 2x^2 + 4x - 2 - 4x^3 + 16x$$

$$\Rightarrow x^4 + 6x^3 + x^2 - 12x - 8 = 0 \stackrel{\odot}{\Rightarrow} (x+1) \cdot (x^3 + 5x^2 - 4x - 8)$$

$$\stackrel{\otimes}{\Rightarrow} (x+1)^2 \cdot (x^2 + 4x - 8) \Rightarrow \begin{cases} (x+1)^2 = 0 \Rightarrow x = -1 \\ x^2 + 4x - 8 = 0 \Rightarrow x = -2 \pm 2\sqrt{3} \end{cases}$$

$$\odot \quad \begin{array}{c|ccc|c} & 1 & 6 & 1 & -12 & -8 \\ -1 & & -1 & -5 & 4 & 8 \\ \hline & 1 & 5 & -4 & -8 & 0 \end{array}$$

$$\otimes \quad \begin{array}{c|ccc|c} & 1 & 5 & -4 & -8 \\ -1 & & -1 & -4 & 8 \\ \hline & 1 & 4 & -8 & 0 \end{array}$$

Ecuaciones irracionales

4.1. Ecuaciones irracionales

① $\sqrt{3x+16} + 1 = 2x$

② $x - \sqrt{2x-1} = 1 - x$

③ $\sqrt{3x+4} + 2x = 4$

④ $x + \sqrt{x} = 30$

⑤ $\sqrt{7-3x} - x = 7$

⑥ $\sqrt{x} + 3 = 2x$

⑦ $1 + \sqrt{x+1} = \frac{x}{3}$

⑧ $\sqrt{x^2 + 3x + 7} = 5$

⑨ $\sqrt{2x+1} + 1 = x$

⑩ $3\sqrt{6x+1} - 5 = 2x$

⑪ $3 - \sqrt{x} = x + 1$

⑫ $\sqrt{2x+5} + 6 = 3x + 3$

⑬ $\sqrt{3x-2} - 4 = 0$

⑭ $\sqrt{x-1} - x + 7 = 0$

⑮ $\sqrt{2x-1} + x = 8$

⑯ $\sqrt{x+1} = x - 1$

⑰ $\sqrt{3x-5} + x = 1$

⑱ $\sqrt{x^2 - 8} = x + 2$

⑲ $\sqrt{2x-1} = x - 2$

⑳ $\sqrt{x^2 + 8} - x = 2$

㉑ $x - \sqrt{x^2 + 15} = -3$

㉒ $\sqrt{x+5} - x = 3$

㉓ $\sqrt{2x-1} + \sqrt{x+4} = 6$

㉔ $\sqrt{4x+5} - \sqrt{3x+1} = 1$

㉕ $\sqrt{3x+10} = 1 + \sqrt{3x+3}$

㉖ $\sqrt{x} + 1 = \sqrt{x+9}$

㉗ $\sqrt{x+4} = 3 - \sqrt{x-1}$

㉘ $2\sqrt{x+4} = \sqrt{5x+4}$

㉙ $\sqrt{x-2} + \sqrt{x+3} = 5$

㉚ $\frac{3}{\sqrt{x}} = \frac{6}{\sqrt{3x+4}}$

㉛ $\sqrt{x+7} - \sqrt{x} = 1$

㉜ $\sqrt{2x+3} - \sqrt{x-2} = 2$

㉝ $\sqrt{x+6} + \sqrt{2-x} = 4$

㉞ $\sqrt{x-1} + \sqrt{x} = 2$

㉟ $\sqrt{x+7} + \sqrt{x} = 7$

4.1. Ecuaciones irracionales

$$(36) \quad \sqrt{x+6} + \sqrt{x} = 3$$

$$(37) \quad \sqrt{x+1} - \sqrt{x-1} = 1$$

$$(38) \quad \sqrt{2x+3} + x - 2 = \sqrt{5x+1}$$

$$(39) \quad \sqrt{x+1} - \sqrt{x-2} = 1$$

$$(40) \quad \sqrt{x+3} - \sqrt{x+2} = 1$$

$$(41) \quad \sqrt{3x+1} + \sqrt{x+1} = 2$$

$$(42) \quad \sqrt{x+3} + \sqrt{x} = 2$$

$$(43) \quad \sqrt{x+4} - \sqrt{x-1} = 1$$

$$(44) \quad \sqrt{2x-1} - \sqrt{x-1} = 1$$

$$(45) \quad \sqrt{2x-3} - \sqrt{x-5} = 2$$

$$(46) \quad \sqrt{x-3} + \sqrt{x+4} = \sqrt{4x+1}$$

$$(47) \quad 2\sqrt{2x-1} = \sqrt{6x-5} + \sqrt{2x-9}$$

$$(48) \quad \sqrt{x^3} - 2\sqrt{x} = \sqrt{x}$$

$$(49) \quad \sqrt{x+6} + \sqrt{x+11} = \sqrt{5-10x}$$

$$(50) \quad \sqrt{2x-1} + \sqrt{2x+1} = \frac{1}{\sqrt{2x-1}}$$

$$(51) \quad \sqrt{9\sqrt{15-x}} = 6\sqrt{2x+3}$$

$$(52) \quad \sqrt{2} + \sqrt{\frac{2^3}{x}} = \sqrt{2x}$$

$$(53) \quad \frac{21}{\sqrt{6x+1}} - \sqrt{6x+1} = 2\sqrt{3x}$$

$$(54) \quad \sqrt{x-1} - 2\sqrt{x+2} = 0$$

$$(55) \quad \sqrt{x - \sqrt{1-x}} + \sqrt{x} = 1$$

$$\textcircled{1} \quad \sqrt{3x+16}+1=2x \implies \sqrt{3x+16}=2x-1 \implies 3x+16=4x^2-4x+1$$

$$\implies 4x^2-7x-15=0 \implies \begin{cases} x = -\frac{5}{4} & \sqrt{-\frac{15}{4}+16}+1 \neq -\frac{5}{4} \\ x = 3 & \sqrt{9+16}+1 = 6 \checkmark \end{cases}$$

$$\textcircled{2} \quad x-\sqrt{2x-1}=1-x \implies 2x-1=\sqrt{2x-1} \implies 4x^2-4x+1=2x-1$$

$$\implies 4x^2-6x+2=0 \implies \begin{cases} x = \frac{1}{2} & \frac{1}{2}-\sqrt{1-1}=1-\frac{1}{2} \checkmark \\ x = 1 & 1-\sqrt{1}=1-1 \checkmark \end{cases}$$

$$\textcircled{3} \quad \sqrt{3x+4}+2x=4 \implies \sqrt{3x+4}=4-2x \implies 3x+4=16+4x^2-16x$$

$$\implies 4x^2-19x+12=0 \implies \begin{cases} x = \frac{3}{4} & \sqrt{\frac{9}{4}+4}+2 \cdot \frac{3}{4} = 4 \checkmark \\ x = 4 & \sqrt{12+4}+8 \neq 4 \end{cases}$$

$$\textcircled{4} \quad x+\sqrt{x}=30 \implies \sqrt{x}=30-x \implies x=900+x^2-60x \implies x^2-61x+900=0$$

$$\implies \begin{cases} x = 25 & 25+\sqrt{25}=30 \checkmark \\ x = 36 & 36+\sqrt{36} \neq 30 \end{cases}$$

$$\textcircled{5} \quad \sqrt{7-3x}-x=7 \implies \sqrt{7-3x}=7+x \implies 7-3x=49+x^2+14x$$

$$\implies x^2+17x+42=0 \implies \begin{cases} x = -3 & \sqrt{7+9}+3 = 7 \checkmark \\ x = -14 & \sqrt{7+42}+14 \neq 7 \end{cases}$$

$$\textcircled{6} \quad \sqrt{x}+3=2x \implies \sqrt{x}=2x-3 \implies x=4x^2+9-12x \implies 4x^2-13x+9=0$$

$$\implies \begin{cases} x = \frac{9}{4} & \sqrt{\frac{9}{4}}+3 = 2 \cdot \frac{9}{4} \checkmark \\ x = 1 & \sqrt{1}+3 \neq 2 \end{cases}$$

$$\textcircled{7} \quad 1+\sqrt{x+1}=\frac{x}{3} \implies \sqrt{x+1}=\frac{x}{3}-1 \implies x+1=\frac{x^2}{9}+1-\frac{2x}{3} \implies x^2-15x=0$$

$$\implies x \cdot (x-15) \implies \begin{cases} x = 0 & 1+\sqrt{1} \neq 0 \\ x = 15 & 1+\sqrt{15+1} = \frac{15}{3} \end{cases}$$

$$\textcircled{8} \quad \sqrt{x^2+3x+7}=5 \implies x^2+3x+7=25 \implies x^2+3x-18=0$$

$$\implies \begin{cases} x = -6 & \sqrt{36-18+7}=5 \checkmark \\ x = 3 & \sqrt{9+9+7}=5 \checkmark \end{cases}$$

$$\textcircled{9} \quad \sqrt{2x+1}+1=x \implies \sqrt{2x+1}=x-1 \implies 2x+1=x^2-2x+1 \implies x^2-4x=0$$

$$\implies x \cdot (x-4)=0 \implies \begin{cases} x = 0 & \sqrt{1}+1 \neq 0 \\ x = 4 & \sqrt{8+1}+1 = 4 \checkmark \end{cases}$$

4.1. Ecuaciones irracionales

$$(10) \quad 3\sqrt{6x+1} - 5 = 2x \implies 3\sqrt{6x+1} = 2x + 5 \implies 9 \cdot (6x+1) = 4x^2 + 25 + 20x$$

$$\implies 4x^2 - 34x + 16 = 0 \implies \begin{cases} x = \frac{1}{2} & 3\sqrt{3+1} - 5 = 2 \cdot \frac{1}{2} \checkmark \\ x = 8 & 3\sqrt{48+1} - 5 = 16 \checkmark \end{cases}$$

$$(11) \quad 3 - \sqrt{x} = x + 1 \implies 2 - x = \sqrt{x} \implies 4 + x^2 - 4x = x \implies x^2 - 5x + 4 = 0$$

$$\implies \begin{cases} x = 1 & 3 - \sqrt{1} = 1 + 1 \checkmark \\ x = 4 & 3 - \sqrt{4} \neq 4 + 1 \end{cases}$$

$$(12) \quad \sqrt{2x+5} + 6 = 3x + 3 \implies \sqrt{2x+5} = 3x - 3 \implies 2x + 5 = 9x^2 - 18x + 9$$

$$\implies 9x^2 - 20x + 4 = 0 \implies \begin{cases} x = \frac{2}{9} & \sqrt{\frac{4}{9} + 5} + 6 \neq \frac{2}{3} + 3 \\ x = 2 & \sqrt{4+5} + 6 = 6 + 3 \checkmark \end{cases}$$

$$(13) \quad \sqrt{3x-2} - 4 = 0 \implies \sqrt{3x-2} = 4 \implies 3x - 2 = 16 \implies 3x = 18$$

$$\implies \boxed{x = 6} \quad \sqrt{18-2} - 4 = 0 \checkmark$$

$$(14) \quad \sqrt{x-1} - x + 7 = 0 \implies \sqrt{x-1} = x - 7 \implies x - 1 = x^2 - 14x + 49$$

$$\implies x^2 - 15x + 50 = 0 \implies \begin{cases} x = 5 & \sqrt{5-1} - 5 + 7 \neq 0 \\ x = 10 & \sqrt{10-1} - 10 + 7 = 0 \checkmark \end{cases}$$

$$(15) \quad \sqrt{2x-1} + x = 8 \implies \sqrt{2x-1} = 8 - x \implies 2x - 1 = 64 + x^2 - 16x$$

$$\implies x^2 - 18x + 65 = 0 \implies \begin{cases} x = 5 & \sqrt{10-1} + 5 = 8 \checkmark \\ x = 13 & \sqrt{26-1} + 13 \neq 8 \end{cases}$$

$$(16) \quad \sqrt{x+1} = x - 1 \implies x + 1 = x^2 - 2x + 1 \implies x^2 - 3x = 0 \implies x \cdot (x - 3) = 0$$

$$\implies \begin{cases} x = 0 & \sqrt{1} \neq 0 - 1 \\ x = 3 & \sqrt{3+1} = 3 - 1 \checkmark \end{cases}$$

$$(17) \quad \sqrt{3x-5} + x = 1 \implies \sqrt{3x-5} = 1 - x \implies 3x - 5 = 1 - 2x + x^2 \implies x^2 - 5x + 6 = 0$$

$$\implies \begin{cases} x = 2 & \sqrt{6-5} + 2 \neq 1 \\ x = 3 & \sqrt{9-5} + 3 \neq 1 \end{cases} \implies \nexists \text{ Solución}$$

$$(18) \quad \sqrt{x^2-8} = x + 2 \implies x^2 - 8 = x^2 + 4x + 4 \implies 4x = -12$$

$$\implies x = -3 \quad \sqrt{9-8} \neq -3 + 2 \implies \nexists \text{ Solución}$$

$$(19) \quad \sqrt{2x-1} = x - 2 \implies 2x - 1 = x^2 - 4x + 4 \implies x^2 - 6x + 5 = 0$$

$$\implies \begin{cases} x = 1 & \sqrt{2-1} \neq 1 - 2 \\ x = 5 & \sqrt{10-1} = 5 - 2 \checkmark \end{cases}$$

- 20 $\sqrt{x^2+8} - x = 2 \Rightarrow \sqrt{x^2+8} = x+2 \Rightarrow x^2+8 = x^2+4x+4 \Rightarrow 4x=4$
 $\Rightarrow \boxed{x=1} \quad \sqrt{1+8} - 1 = 2 \checkmark$
- 21 $x - \sqrt{x^2+15} = -3 \Rightarrow x+3 = \sqrt{x^2+15} \Rightarrow x^2+6x+9 = x^2+15 \Rightarrow 6x=6$
 $\Rightarrow \boxed{x=1} \quad 1 - \sqrt{1+15} = -3 \checkmark$
- 22 $\sqrt{x+5} - x = 3 \Rightarrow \sqrt{x+5} = 3+x \Rightarrow x+5 = 9+x^2+6x \Rightarrow x^2+5x+4=0$
 $\Rightarrow \begin{cases} \boxed{x=-1} & \sqrt{-1+5} + 1 = 3 \checkmark \\ \cancel{x=-4} & \sqrt{-4+5} + 4 \neq 3 \end{cases}$
- 23 $\sqrt{2x-1} + \sqrt{x+4} = 6 \Rightarrow \sqrt{2x-1} = 6 - \sqrt{x+4}$
 $\Rightarrow 2x-1 = 36 + x+4 - 12\sqrt{x+4} \Rightarrow 12\sqrt{x+4} = 41-x$
 $\Rightarrow 144 \cdot (x+4) = 1681 + x^2 - 82x \Rightarrow x^2 - 226x + 1105 = 0$
 $\Rightarrow \begin{cases} \boxed{x=5} & \sqrt{10-1} + \sqrt{5+4} = 6 \checkmark \\ \cancel{x=221} & \sqrt{442-1} + \sqrt{221+4} \neq 6 \end{cases}$
- 24 $\sqrt{4x+5} - \sqrt{3x+1} = 1 \Rightarrow \sqrt{4x+5} = 1 + \sqrt{3x+1}$
 $\Rightarrow 4x+5 = 1 + 3x+1 + 2\sqrt{3x+1} \Rightarrow x+3 = 2\sqrt{3x+1}$
 $\Rightarrow x^2+6x+9 = 4 \cdot (3x+1) \Rightarrow x^2-6x+5=0 \Rightarrow \begin{cases} \boxed{x=1} & \sqrt{4+5} - \sqrt{3+1} = 1 \checkmark \\ \boxed{x=5} & \sqrt{20+5} - \sqrt{15+1} = 1 \checkmark \end{cases}$
- 25 $\sqrt{3x+10} = 1 + \sqrt{3x+3} \Rightarrow 3x+10 = 1 + 3x+3 + 2\sqrt{3x+3}$
 $\Rightarrow 6 = 2\sqrt{3x+3} \Rightarrow 36 = 4 \cdot (3x+3)$
 $\Rightarrow 12x = 24 \Rightarrow \boxed{x=2} \quad \sqrt{6+10} = 1 + \sqrt{6+3} \checkmark$
- 26 $\sqrt{x} + 1 = \sqrt{x+9} \Rightarrow x+1 + 2\sqrt{x} = x+9 \Rightarrow 2\sqrt{x} = 8 \Rightarrow 4x = 64$
 $\Rightarrow \boxed{x=16} \quad \sqrt{16} + 1 = \sqrt{16+9} \checkmark$
- 27 $\sqrt{x+4} = 3 - \sqrt{x-1} \Rightarrow x+4 = 9 + x-1 - 6\sqrt{x-1} \Rightarrow 4 = 6\sqrt{x-1}$
 $\Rightarrow 16 = 36 \cdot (x-1) \Rightarrow 36x = 52 \Rightarrow \boxed{x = \frac{13}{9}} \quad \sqrt{\frac{13}{9}+4} = 3 - \sqrt{\frac{13}{9}-1} \checkmark$
- 28 $2\sqrt{x+4} = \sqrt{5x+4} \Rightarrow 4 \cdot (x+4) = 5x+4$
 $\Rightarrow \boxed{x=12} \Rightarrow 2\sqrt{12+4} = \sqrt{60+4} \checkmark$
- 29 $\sqrt{x-2} + \sqrt{x+3} = 5 \Rightarrow \sqrt{x-2} = 5 - \sqrt{x+3}$
 $\Rightarrow x-2 = 25 + x+3 - 10\sqrt{x+3} \Rightarrow 30 = 10\sqrt{x+3} \Rightarrow 3 = \sqrt{x+3}$
 $\Rightarrow 9 = x+3 \Rightarrow \boxed{x=6} \quad \sqrt{6-2} + \sqrt{6+3} = 5 \checkmark$

4.1. Ecuaciones irracionales

$$\begin{aligned} \textcircled{30} \quad \frac{3}{\sqrt{x}} = \frac{6}{\sqrt{3x+4}} &\Rightarrow \frac{9}{x} = \frac{36}{3x+4} \Rightarrow 27x+36 = 36x \Rightarrow 9x = 36 \\ &\Rightarrow \boxed{x=4} \quad \frac{3}{\sqrt{4}} = \frac{6}{\sqrt{12+4}} \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{31} \quad \sqrt{x+7} - \sqrt{x} = 1 &\Rightarrow \sqrt{x+7} = 1 + \sqrt{x} \Rightarrow x+7 = 1 + x + 2\sqrt{x} \\ &\Rightarrow 6 = 2\sqrt{x} \Rightarrow 36 = 4x \Rightarrow \boxed{x=9} \quad \sqrt{9+7} - \sqrt{9} = 1 \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{32} \quad \sqrt{2x+3} - \sqrt{x-2} = 2 &\Rightarrow \sqrt{2x+3} = 2 + \sqrt{x-2} \Rightarrow 2x+3 = 4 + x - 2 + 4\sqrt{x-2} \\ &\Rightarrow x+1 = 4\sqrt{x-2} \Rightarrow x^2 + 2x + 1 = 16 \cdot (x-2) \Rightarrow x^2 - 14x + 33 = 0 \\ &\Rightarrow \begin{cases} \boxed{x=3} & \sqrt{6+3} - \sqrt{3-2} = 2 \checkmark \\ \boxed{x=11} & \sqrt{22+3} - \sqrt{11-2} = 2 \checkmark \end{cases} \end{aligned}$$

$$\begin{aligned} \textcircled{33} \quad \sqrt{x+6} + \sqrt{2-x} = 4 &\Rightarrow \sqrt{x+6} = 4 - \sqrt{2-x} \Rightarrow x+6 = 16 + 2 - x - 8\sqrt{2-x} \\ &\Rightarrow 2x-12 = 8\sqrt{2-x} \Rightarrow 4x^2 - 48x + 144 = 64 \cdot (2-x) \Rightarrow 4x^2 + 16x + 16 = 0 \\ &\Rightarrow \boxed{x=-2} \quad \sqrt{-2+6} + \sqrt{2+2} = 4 \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{34} \quad \sqrt{x-1} + \sqrt{x} = 2 &\Rightarrow \sqrt{x-1} = 2 - \sqrt{x} \Rightarrow x-1 = 4 + x - 4\sqrt{x} \Rightarrow 5 = 4\sqrt{x} \\ &\Rightarrow 25 = 16x \Rightarrow \boxed{x = \frac{25}{16}} \quad \sqrt{\frac{25}{16}} - 1 + \sqrt{\frac{25}{16}} = 2 \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{35} \quad \sqrt{x+7} + \sqrt{x} = 7 &\Rightarrow \sqrt{x+7} = 7 - \sqrt{x} \Rightarrow x+7 = 49 + x - 14\sqrt{x} \\ &\Rightarrow 42 = 14\sqrt{x} \Rightarrow 3 = \sqrt{x} \Rightarrow \boxed{x=9} \quad \sqrt{9+7} + \sqrt{9} = 7 \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{36} \quad \sqrt{x+6} + \sqrt{x} = 3 &\Rightarrow \sqrt{x+6} = 3 - \sqrt{x} \Rightarrow x+6 = 9 + x - 6\sqrt{x} \\ &\Rightarrow 3 = 6\sqrt{x} \Rightarrow 9 = 36x \Rightarrow \boxed{x = \frac{1}{4}} \quad \sqrt{\frac{1}{4}+6} + \sqrt{\frac{1}{4}} = 3 \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{37} \quad \sqrt{x+1} - \sqrt{x-1} = 1 &\Rightarrow \sqrt{x+1} = 1 + \sqrt{x-1} \Rightarrow x+1 = 1 + x - 1 + 2\sqrt{x-1} \\ &\Rightarrow 1 = 2\sqrt{x-1} \Rightarrow 1 = 4 \cdot (x-1) \Rightarrow \boxed{x = \frac{5}{4}} \quad \sqrt{\frac{5}{4}+1} - \sqrt{\frac{5}{4}-1} = 1 \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{38} \quad \sqrt{2x+3} + x - 2 = \sqrt{5x+1} &\Rightarrow \sqrt{2x+3} - \sqrt{5x+1} = 2 - x \\ &\Rightarrow 2x+3 + 5x+1 - 2\sqrt{(2x+3) \cdot (5x+1)} = 4 + x^2 - 4x \\ &\Rightarrow x^2 - 11x = -2\sqrt{10x^2 + 17x + 3} \Rightarrow x^4 + 121x^2 - 22x^3 = 4 \cdot (10x^2 + 17x + 3) \\ &\Rightarrow x^4 - 22x^3 + 81x^2 - 68x - 12 = 0 \Rightarrow \textcircled{\bullet} \boxed{x=3} \quad \sqrt{6+3} + 3 - 2 = \sqrt{15+1} \checkmark \end{aligned}$$

$$\textcircled{\bullet} \quad \begin{array}{c|ccc|c} & 1 & -22 & 81 & -68 & -12 \\ 3 & & 3 & -57 & 72 & 12 \\ \hline & 1 & -19 & 24 & 4 & 0 \end{array}$$

$$\begin{aligned} \textcircled{39} \quad \sqrt{x+1} - \sqrt{x-2} = 1 &\implies \sqrt{x+1} = 1 + \sqrt{x-2} \implies x+1 = 1 + x-2 + 2\sqrt{x-2} \\ &\implies 1 = \sqrt{x-2} \implies 1 = x-2 \implies \boxed{x=3} \quad \sqrt{3+1} - \sqrt{3-2} = 1 \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{40} \quad \sqrt{x+3} - \sqrt{x+2} = 1 &\implies \sqrt{x+3} = 1 + \sqrt{x+2} \implies x+3 = 1 + x+2 + 2\sqrt{x+2} \\ &\implies 0 = \sqrt{x+2} \implies x+2 = 0 \implies \boxed{x=-2} \quad \sqrt{-2+3} - \sqrt{-2+2} = 1 \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{41} \quad \sqrt{3x+1} + \sqrt{x+1} = 2 &\implies \sqrt{3x+1} = 2 - \sqrt{x+1} \implies 3x+1 = 4 + x+1 - 4\sqrt{x+1} \\ &\implies 4\sqrt{x+1} = 4 - 2x \implies 16 \cdot (x+1) = 16 + 4x^2 - 16x \implies 4x^2 - 32x = 0 \\ &\implies 4x \cdot (x-8) = 0 \implies \begin{cases} \boxed{x=0} & \sqrt{1} + \sqrt{1} = 2 \checkmark \\ \cancel{x=8} & \sqrt{24+1} + \sqrt{8+1} \neq 2 \end{cases} \end{aligned}$$

$$\begin{aligned} \textcircled{42} \quad \sqrt{x+3} + \sqrt{x} = 2 &\implies \sqrt{x+3} = 2 - \sqrt{x} \implies x+3 = 4 + x - 4\sqrt{x} \\ &\implies 4\sqrt{x} = 1 \implies 16x = 1 \implies \boxed{x = \frac{1}{16}} \quad \sqrt{\frac{1}{16}+3} + \sqrt{\frac{1}{16}} = 2 \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{43} \quad \sqrt{x+4} - \sqrt{x-1} = 1 &\implies \sqrt{x+4} = 1 + \sqrt{x-1} \implies x+4 = 1 + x-1 + 2\sqrt{x-1} \\ &\implies 2 = \sqrt{x-1} \implies 4 = x-1 \implies \boxed{x=5} \quad \sqrt{5+4} - \sqrt{5-1} = 1 \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{44} \quad \sqrt{2x-1} - \sqrt{x-1} = 1 &\implies \sqrt{2x-1} = 1 + \sqrt{x-1} \implies 2x-1 = 1 + x-1 + 2\sqrt{x-1} \\ &\implies x-1 = 2\sqrt{x-1} \implies x^2 - 2x + 1 = 4 \cdot (x-1) \implies x^2 - 6x + 5 = 0 \\ &\implies \begin{cases} \boxed{x=1} & \sqrt{2-1} - \sqrt{1-1} = 1 \checkmark \\ \boxed{x=5} & \sqrt{10-1} - \sqrt{5-1} = 1 \checkmark \end{cases} \end{aligned}$$

$$\begin{aligned} \textcircled{45} \quad \sqrt{2x-3} - \sqrt{x-5} = 2 &\implies \sqrt{2x-3} = 2 + \sqrt{x-5} \implies 2x-3 = 4 + x-5 + 4\sqrt{x-5} \\ &\implies x-2 = 4\sqrt{x-5} \implies x^2 - 4x + 4 = 16 \cdot (x-5) \implies x^2 - 20x + 84 = 0 \\ &\implies \begin{cases} \boxed{x=6} & \sqrt{12-3} - \sqrt{6-5} = 2 \checkmark \\ \boxed{x=14} & \sqrt{28-3} - \sqrt{14-5} = 2 \checkmark \end{cases} \end{aligned}$$

$$\begin{aligned} \textcircled{46} \quad \sqrt{x-3} + \sqrt{x+4} = \sqrt{4x+1} &\implies x-3 + x+4 + 2\sqrt{(x-3) \cdot (x+4)} = 4x+1 \\ &\implies 2\sqrt{x^2+x-12} = 2x \implies x^2+x-12 = x^2 \\ &\implies \boxed{x=12} \quad \sqrt{12-3} + \sqrt{12+4} = \sqrt{48+1} \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{47} \quad 2\sqrt{2x-1} = \sqrt{6x-5} + \sqrt{2x-9} &\implies 4 \cdot (2x-1) = 6x-5 + 2x-9 + 2\sqrt{(6x-5) \cdot (2x-9)} \\ &\implies 5 = \sqrt{12x^2 - 64x + 45} \implies 25 = 12x^2 - 64x + 45 \implies 12x^2 - 64x + 20 = 0 \\ &\implies \begin{cases} \cancel{x = \frac{1}{3}} & 2\sqrt{\frac{2}{3}-1} = \sqrt{6 \cdot \frac{1}{3}-5} + \sqrt{2 \cdot \frac{1}{3}-9} \\ \boxed{x=5} & 2\sqrt{10-1} = \sqrt{30-5} + \sqrt{10-9} \checkmark \end{cases} \end{aligned}$$

4.1. Ecuaciones irracionales

$$(48) \quad \sqrt{x^3} - 2\sqrt{x} = \sqrt{x} \implies x^3 + 4x - 4\sqrt{x^4} = x \implies x^3 - 4x^2 + 3x = 0$$

$$\implies x \cdot (x^2 - 4x + 3) = 0 \implies \begin{cases} \boxed{x=0} & \sqrt{0} - 2\sqrt{0} = \sqrt{0} \checkmark \\ \cancel{x=1} & \sqrt{1} - 2\sqrt{1} \neq \sqrt{1} \\ \boxed{x=3} & \sqrt{27} - 2\sqrt{3} = \sqrt{3} \checkmark \end{cases}$$

$$(49) \quad \sqrt{x+6} + \sqrt{x+11} = \sqrt{5-10x} \implies x+6 + x+11 + 2\sqrt{(x+6) \cdot (x+11)} = 5-10x$$

$$\implies 2\sqrt{x^2+17x+66} = -12x-12 \implies \sqrt{x^2+17x+66} = -6x-6$$

$$\implies x^2+17x+66 = 36x^2+72x+36 \implies 35x^2+55x-30=0$$

$$\implies \begin{cases} \boxed{x=-2} & \sqrt{-2+6} + \sqrt{-2+11} = \sqrt{5+20} \checkmark \\ \cancel{x=\frac{3}{7}} & \sqrt{\frac{3}{7}+6} + \sqrt{\frac{3}{7}+11} \neq \sqrt{5-\frac{30}{7}} \end{cases}$$

$$(50) \quad \sqrt{2x-1} + \sqrt{2x+1} = \frac{1}{\sqrt{2x-1}} \implies 2x-1 + \sqrt{(2x+1) \cdot (2x-1)} = 1$$

$$\implies 2x-1 + \sqrt{4x^2-1} = 1 \implies \sqrt{4x^2-1} = 2-2x \implies 4x^2-1 = 4+4x^2-8x$$

$$\implies \boxed{x=\frac{5}{8}} \quad \sqrt{\frac{5}{4}-1} + \sqrt{\frac{5}{4}+1} = \frac{1}{\sqrt{\frac{5}{4}-1}} \checkmark$$

$$(51) \quad \sqrt{9\sqrt{15-x}} = 6\sqrt{2x+3} \implies 9\sqrt{15-x} = 36 \cdot (2x+3) \implies \sqrt{15-x} = 4 \cdot (2x+3)$$

$$\implies 15-x = 16 \cdot (2x+3)^2 \implies 15-x = 16 \cdot (4x^2+9+12x) \implies 64x^2+193x+129=0$$

$$\implies \begin{cases} \boxed{x=-1} & \sqrt{9 \cdot 4} = 6\sqrt{1} \checkmark \\ \cancel{x=-\frac{129}{64}} & \sqrt{9\sqrt{15+\frac{129}{64}}} \neq 6\sqrt{\frac{129}{32}+3} \end{cases}$$

$$(52) \quad \sqrt{2} + \sqrt{\frac{2^3}{x}} = \sqrt{2x} \implies \sqrt{\frac{2^3}{x}} = \sqrt{2x} - \sqrt{2} \implies \frac{8}{x} = 2x+2-2\sqrt{4x}$$

$$\implies \frac{2x^2+2x-8}{x} = 2\sqrt{4x} \implies \frac{x^2+x-4}{x} = \sqrt{4x}$$

$$\Rightarrow \frac{x^4+x^2+16+2x^3-8x^2-8x}{x^2} = 4x \implies \frac{x^4+x^2+16+2x^3-8x^2-8x}{\cancel{x^2}} = \frac{4x^3}{\cancel{x^2}}$$

$$\implies x^4-2x^3-7x^2-8x+16=0 \stackrel{\odot}{\implies} (x-1) \cdot (x-4) \cdot (x^2+3x+4)=0$$

$$\implies \begin{cases} \cancel{x=1} & \sqrt{2} + \sqrt{\frac{8}{1}} \neq \sqrt{2} \\ \boxed{x=4} & \sqrt{2} + \sqrt{\frac{8}{4}} = \sqrt{8} \checkmark \end{cases}$$

$$\odot \quad \begin{array}{c|cccc|c} 1 & 1 & -2 & -7 & -8 & 16 \\ & & 1 & -1 & -8 & -16 \\ \hline & 1 & -1 & -8 & -16 & 0 \end{array} \quad \begin{array}{c|ccc|c} 4 & 1 & -1 & -8 & -16 \\ & & 4 & 12 & 16 \\ \hline & 1 & 3 & 4 & 0 \end{array}$$

$$(53) \quad \frac{21}{\sqrt{6x+1}} - \sqrt{6x+1} = 2\sqrt{3x} \implies 21 - (6x+1) = 2\sqrt{3x \cdot (6x+1)}$$

$$\Rightarrow 20-6x = 2\sqrt{18x^2+3x} \Rightarrow 10-3x = \sqrt{18x^2+3x} \Rightarrow 100+9x^2-60x = 18x^2+3x$$

$$\Rightarrow 9x^2 + 63x - 100 = 0 \Rightarrow \begin{cases} \boxed{x = \frac{4}{3}} & \frac{21}{\sqrt{8+1}} - \sqrt{8+1} = 2\sqrt{4} \checkmark \\ \cancel{x = -\frac{25}{3}} & \frac{21}{\sqrt{-50+1}} \xrightarrow{\#} \dots \end{cases}$$

$$\textcircled{54} \quad \sqrt{x-1} - 2\sqrt{x+2} = 0 \Rightarrow \sqrt{x-1} = 2\sqrt{x+2} \Rightarrow x-1 = 4 \cdot (x+2)$$

$$\Rightarrow x-1 = 4x+8 \Rightarrow 3x = -9 \Rightarrow \cancel{x = -3} \quad \sqrt{-4} \xrightarrow{\#} \dots$$

$$\textcircled{55} \quad \sqrt{x - \sqrt{1-x}} + \sqrt{x} = 1 \Rightarrow \sqrt{x - \sqrt{1-x}} = 1 - \sqrt{x} \Rightarrow x - \sqrt{1-x} = 1 + x - 2\sqrt{x}$$

$$\Rightarrow -\sqrt{1-x} = 1 - 2\sqrt{x} \Rightarrow \cancel{1} - x = \cancel{1} + 4x - 4\sqrt{x} \Rightarrow -5x = -4\sqrt{x}$$

$$\Rightarrow 25x^2 = 16x \Rightarrow 25x^2 - 16x = 0 \Rightarrow x \cdot (25x - 16) = 0$$

$$\Rightarrow \begin{cases} \cancel{x = 0} & \sqrt{-1} \xrightarrow{\#} \dots \\ 25x - 16 = 0 \Rightarrow \boxed{x = \frac{16}{25}} & \sqrt{\frac{16}{25} - \sqrt{\frac{9}{25}}} + \sqrt{\frac{16}{25}} = \sqrt{\frac{16}{25} - \frac{3}{5}} + \frac{4}{5} = \frac{1}{5} + \frac{4}{5} = 1 \checkmark \end{cases}$$

Ecuaciones no lineales

5.1. Sistemas de ecuaciones no lineales

$$\textcircled{1} \begin{cases} xy - 2x = -9 \\ \frac{x+y}{y} = 2y \end{cases}$$

$$\textcircled{2} \begin{cases} 6x + 6y = 5 \\ x^2 - y^2 = \frac{5}{36} \end{cases}$$

$$\textcircled{3} \begin{cases} y^2 - x^2 = 2 \cdot (x + y) \\ \sqrt{x+y+4} + x = y + 2 \end{cases}$$

$$\textcircled{4} \begin{cases} \frac{x+1}{2} = y - 3 \\ 2x^2 = y^2 - 7 \end{cases}$$

$$\textcircled{5} \begin{cases} \frac{x}{y+1} = 2 \\ 3xy - 2x = 100 \end{cases}$$

$$\textcircled{6} \begin{cases} 5x + 6y = -1 \\ \frac{3}{x} + \frac{4}{y} = -1 \end{cases}$$

$$\textcircled{7} \begin{cases} \frac{x+2}{y-4} = 2 \\ (x-3) \cdot (y+1) = 50 \end{cases}$$

$$\textcircled{8} \begin{cases} x + y = 24 \\ x^2 + y^2 = 290 \end{cases}$$

$$\textcircled{9} \begin{cases} 2x + y = 3 \\ x^2 + y^2 = 9 \end{cases}$$

$$\textcircled{10} \begin{cases} y + 3 = 3x \\ x^2 + y^2 = 13 \end{cases}$$

$$\textcircled{11} \begin{cases} y + 5 = 3x \\ x - 2y^2 = 0 \end{cases}$$

$$\textcircled{12} \begin{cases} x - \frac{3}{4}y = 0 \\ x^2 + y^2 = 25 \end{cases}$$

$$\textcircled{13} \begin{cases} x + y = 5 \\ x^2 + 3xy = 22 \end{cases}$$

$$\textcircled{14} \begin{cases} y - x = 1 \\ 4x^2 - xy = 2 \cdot (x + y) \end{cases}$$

$$\textcircled{15} \begin{cases} x + y = 5 \\ x^2 - xy + y^2 = 7 \end{cases}$$

$$\textcircled{16} \begin{cases} y = 1 + 2x \\ x^2 + y^2 + 6x = 16 \end{cases}$$

$$\textcircled{17} \begin{cases} x = 3y - 1 \\ \frac{1}{x} - \frac{1}{y} = \frac{-1}{2} \end{cases}$$

$$\textcircled{18} \begin{cases} x + y = 6 \\ xy = 8 \end{cases}$$

$$\textcircled{19} \begin{cases} x + y = 6 \\ xy = 9 \end{cases}$$

$$\textcircled{20} \begin{cases} x - y = 1 \\ x^2 - y^2 = 17 \end{cases}$$

$$\textcircled{21} \begin{cases} x + y = 3 \\ \frac{1}{x} + \frac{1}{y} = \frac{3}{2} \end{cases}$$

$$\textcircled{22} \begin{cases} 3x - 8y = 5 \\ 4xy - 6y = 3 \end{cases}$$

$$\textcircled{23} \begin{cases} 3x - 2y = 1 \\ 3xy - 4y^2 = 0 \end{cases}$$

$$\textcircled{24} \begin{cases} x - y = 5 \\ 2 \cdot (x + 2y)^2 - (2x + y)^2 = -14 \end{cases}$$

$$\textcircled{25} \begin{cases} x + y = 2 \\ \frac{x-y}{x+y} + \frac{x+y}{x-y} = \frac{5}{2} \end{cases}$$

$$\textcircled{26} \begin{cases} 7x - 2y = 1 \\ \frac{x^2 - 3x + 4}{y^2 - 2y + 3} = \frac{1}{3} \end{cases}$$

$$\textcircled{27} \begin{cases} 3x + 1 = 2x^2 - y^2 \\ 3 - \frac{2x-1}{x+1} = \frac{y+3}{y+1} \end{cases}$$

$$\textcircled{28} \begin{cases} xy = 30 \\ x^2 + y^2 = 61 \end{cases}$$

$$\textcircled{29} \begin{cases} xy + 12 = 0 \\ x^2 + y^2 = 25 \end{cases}$$

$$\textcircled{30} \begin{cases} xy + 3 = 0 \\ 2x^2 - 5y^2 = 13 \end{cases}$$

5.1. Sistemas de ecuaciones no lineales

$$31 \quad \begin{cases} xy = 6 \\ x^2 + xy + y^2 = 19 \end{cases}$$

$$38 \quad \begin{cases} y^2 + xy = 45 \\ x^2 + 2xy = 56 \end{cases}$$

$$43 \quad \begin{cases} x^2 + y^2 = 25 + 2xy \\ x^2 + 2xy + y^2 = 169 \end{cases}$$

$$32 \quad \begin{cases} x^2 - yx + y^2 = 7 \\ x^2 + y^2 = 13 \end{cases}$$

$$39 \quad \begin{cases} xy = 6 \\ \frac{1}{x} + \frac{1}{y} = \frac{5}{6} \end{cases}$$

$$44 \quad \begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{5}{6} \\ \frac{1}{x} - \frac{1}{y} = \frac{1}{6} \end{cases}$$

$$33 \quad \begin{cases} x^2 + 2y^2 = 22 \\ 2x^2 - y^2 = -1 \end{cases}$$

$$40 \quad \begin{cases} \frac{1}{x} - \frac{1}{y} = 1 \\ \frac{1}{x^2} + \frac{1}{y^2} = 13 \end{cases}$$

$$45 \quad \begin{cases} \frac{2}{x} + \frac{3}{y} = \frac{17}{12} \\ \frac{1}{x} - \frac{2}{y} = \frac{-1}{6} \end{cases}$$

$$34 \quad \begin{cases} 2x^2 - 3y^2 = -6 \\ 4x^2 - y^2 = 8 \end{cases}$$

$$35 \quad \begin{cases} y^2 = 16 + 4x \\ x^2 + y^2 + 9x + 14 = 0 \end{cases}$$

$$41 \quad \begin{cases} x + \frac{2}{y} = 1 \\ y + \frac{1}{x} = 6 \end{cases}$$

$$46 \quad \begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{9}{20} \\ x^2y + xy^2 = 180 \end{cases}$$

$$36 \quad \begin{cases} y^2 = 6x \\ x^2 - y^2 + 8 = 0 \end{cases}$$

$$42 \quad \begin{cases} y + \frac{x}{y} = \frac{21}{2} \\ x - \frac{x}{y} = \frac{9}{2} \end{cases}$$

$$47 \quad \begin{cases} x - y = 105 \\ \sqrt{x} + \sqrt{y} = 15 \end{cases}$$

$$37 \quad \begin{cases} x^2 + xy = 77 \\ xy + y^2 = 44 \end{cases}$$

$$48 \quad \begin{cases} x + y = 5\sqrt{y} \\ \sqrt{x} - \sqrt{y} = 1 \end{cases}$$

$$\textcircled{1} \begin{cases} xy - 2x = -9 \implies x \cdot (y - 2) = -9 \implies x = -\frac{9}{y-2} \\ \frac{x+y}{y} = 2y \implies x+y = 2y^2 \xrightarrow{x=-\frac{9}{y-2}} -\frac{9}{y-2} + y = 2y^2 \\ \implies -9 + y \cdot (y-2) = 2y^2 \cdot (y-2) \implies 2y^3 - 5y^2 + 2y + 9 = 0 \end{cases}$$

$$\xrightarrow{\odot} (y+1) \cdot (2y^2 - 7y + 9) \implies \begin{cases} y-1 = 0 \implies \boxed{y=1} \implies \boxed{x=9} \\ 2y \cdot 2 - 7y + 9 = 0 \implies \nexists \text{ Solución} \end{cases}$$

$$\odot \quad -1 \quad \left| \begin{array}{cccc} 2 & -5 & 2 & 9 \\ & -2 & 7 & -9 \\ \hline 2 & -7 & 9 & 0 \end{array} \right.$$

$$\textcircled{2} \begin{cases} 6x + 6y = 5 \implies y = \frac{5}{6} - x \\ x^2 - y^2 = \frac{5}{36} \xrightarrow{y=\frac{5}{6}-x} x^2 - \left(\frac{5}{6} - x\right)^2 = \frac{5}{36} \implies \cancel{x^2} - \frac{25}{36} - \cancel{x^2} + \frac{5x}{3} = \frac{5}{36} \\ \implies \frac{5x}{3} = \frac{5}{6} \implies \boxed{x = 1/2} \implies \boxed{y = 1/3} \end{cases}$$

$$\textcircled{3} \begin{cases} y^2 - x^2 = 2 \cdot (x+y) \implies (\cancel{y+x}) \cdot (y-x) = 2 \cdot (\cancel{x+y}) \implies y = x+2 \\ \sqrt{x+y+4} + x = y+2 \implies \sqrt{x+x+2+4} + \cancel{x} = \cancel{x} + 2 + 2 \implies \sqrt{2x+6} = 4 \\ \implies 2x+6 = 16 \implies \boxed{x=5} \implies \boxed{y=7} \end{cases}$$

$$\textcircled{4} \begin{cases} \frac{x+1}{2} = y-3 \implies x+1 = 2y-6 \implies x = 2y-7 \\ 2x^2 = y^2 - 7 \xrightarrow{x=2y-7} 2 \cdot (2y-7)^2 = y^2 - 7 \implies 2 \cdot (4y^2 + 49 - 28y) = y^2 - 7 \\ \implies 7y^2 - 56y + 105 = 0 \implies \begin{cases} \boxed{y=3} \implies \boxed{x=-1} \\ \boxed{y=5} \implies \boxed{x=3} \end{cases} \end{cases}$$

$$\textcircled{5} \begin{cases} \frac{x}{y+1} = 2 \implies x = 2y+2 \\ 3xy - 2x = 100 \implies x \cdot (3y-2) = 100 \xrightarrow{x=2y+2} (2y+2) \cdot (3y-2) = 100 \\ \implies 6y^2 - 4y + 6y - 4 = 100 \implies 6y^2 + 2y - 104 = 0 \\ \implies \begin{cases} \boxed{y = -13/3} \implies \boxed{x = -20/3} \\ \boxed{y = 4} \implies \boxed{x = 10} \end{cases} \end{cases}$$

$$\textcircled{6} \begin{cases} 5x + 6y = -1 \implies y = -\frac{1+5x}{6} \\ \frac{3}{x} + \frac{4}{y} = -1 \xrightarrow{y=-\frac{1+5x}{6}} \frac{3}{x} - \frac{4}{\frac{1+5x}{6}} = -1 \implies \frac{3}{x} - \frac{24}{1+5x} = -1 \\ \implies \frac{3 \cdot (1+5x) - 24x}{\cancel{x \cdot (1+5x)}} = \frac{-x \cdot (1+5x)}{\cancel{x \cdot (1+5x)}} \\ \implies 3 + 15x - 24x = -x - 5x^2 \implies 5x^2 - 8x + 3 = 0 \end{cases}$$

5.1. Sistemas de ecuaciones no lineales

$$\Rightarrow \begin{cases} \boxed{x = 3/5} \Rightarrow \boxed{y = -2/3} \\ \boxed{x = 1} \Rightarrow \boxed{y = -1} \end{cases}$$

$$\begin{aligned} \textcircled{7} \quad & \begin{cases} \frac{x+2}{y-4} = 2 \Rightarrow x+2 = 2y-8 \Rightarrow x = 2y-10 \\ (x-3) \cdot (y+1) = 50 \xrightarrow{x=2y-10} (2y-10-3) \cdot (y+1) = 50 \end{cases} \\ & \Rightarrow (2y-13) \cdot (y+1) = 50 \Rightarrow 2y^2 - 11y - 63 = 0 \\ & \Rightarrow \begin{cases} \boxed{y = -7/2} \Rightarrow \boxed{x = -17} \\ \boxed{y = 9} \Rightarrow \boxed{x = 8} \end{cases} \end{aligned}$$

$$\begin{aligned} \textcircled{8} \quad & \begin{cases} x+y = 24 \Rightarrow y = 24-x \\ x^2 + y^2 = 290 \xrightarrow{y=24-x} x^2 + (24-x)^2 = 290 \Rightarrow x^2 + 576 + x^2 - 48x = 290 \end{cases} \\ & \Rightarrow 2x^2 - 48x + 286 = 0 \Rightarrow \begin{cases} \boxed{x = 11} \Rightarrow \boxed{y = 13} \\ \boxed{x = 13} \Rightarrow \boxed{y = 11} \end{cases} \end{aligned}$$

$$\begin{aligned} \textcircled{9} \quad & \begin{cases} 2x+y = 3 \Rightarrow y = 3-2x \\ x^2 + y^2 = 9 \xrightarrow{y=3-2x} x^2 + (3-2x)^2 = 9 \Rightarrow x^2 + 9 + 4x^2 - 12x = 9 \end{cases} \\ & \Rightarrow 5x^2 - 12x = 0 \Rightarrow x \cdot (5x - 12) = 0 \\ & \Rightarrow \begin{cases} \boxed{x = 0} \Rightarrow \boxed{y = 3} \\ 5x - 12 = 0 \Rightarrow \boxed{x = 12/5} \Rightarrow \boxed{y = -9/5} \end{cases} \end{aligned}$$

$$\begin{aligned} \textcircled{10} \quad & \begin{cases} y+3 = 3x \Rightarrow y = 3x-3 \\ x^2 + y^2 = 13 \xrightarrow{y=3x-3} x^2 + (3x-3)^2 = 13 \Rightarrow x^2 + 9x^2 + 9 - 18x = 13 \end{cases} \\ & \Rightarrow 10x^2 - 18x - 4 = 0 \Rightarrow \begin{cases} \boxed{x = -1/5} \Rightarrow \boxed{y = -18/5} \\ \boxed{x = 2} \Rightarrow \boxed{y = 3} \end{cases} \end{aligned}$$

$$\begin{aligned} \textcircled{11} \quad & \begin{cases} y+5 = 3x \Rightarrow y = 3x-5 \\ x-2y^2 = 0 \xrightarrow{y=3x-5} x-2 \cdot (3x-5)^2 = 0 \Rightarrow x-2 \cdot (9x^2+25-30x) = 0 \end{cases} \\ & \Rightarrow 18x^2 - 61x + 50 = 0 \Rightarrow \begin{cases} \boxed{x = 25/18} \Rightarrow \boxed{y = -5/6} \\ \boxed{x = 2} \Rightarrow \boxed{y = 1} \end{cases} \end{aligned}$$

$$\begin{aligned} \textcircled{12} \quad & \begin{cases} x - \frac{3}{4}y = 0 \Rightarrow x = \frac{3y}{4} \\ x^2 + y^2 = 25 \xrightarrow{x=3y/4} \left(\frac{3y}{4}\right)^2 + y^2 = 25 \Rightarrow \frac{9y^2}{16} + y^2 = 25 \Rightarrow 25y^2 = 400 \end{cases} \\ & \Rightarrow y^2 = 16 \Rightarrow \begin{cases} \boxed{y = -4} \Rightarrow \boxed{x = -3} \\ \boxed{y = 4} \Rightarrow \boxed{x = 3} \end{cases} \end{aligned}$$

- 13 $\begin{cases} x + y = 5 \implies y = 5 - x \\ x^2 + 3xy = 22 \xrightarrow{y=5-x} x^2 + 3x \cdot (5 - x) = 22 \implies x^2 + 15x - 3x^2 = 22 \end{cases}$
 $\implies 2x^2 - 15x + 22 = 0 \implies \begin{cases} \boxed{x = 2} \implies \boxed{y = 3} \\ \boxed{x = 11/2} \implies \boxed{y = -1/2} \end{cases}$
- 14 $\begin{cases} y - x = 1 \implies y = 1 + x \\ 4x^2 - xy = 2 \cdot (x + y) \xrightarrow{y=1+x} 4x^2 - x \cdot (1 + x) = 2 \cdot (x + 1 + x) \implies 4x^2 - x - x^2 = 4x + 2 \end{cases}$
 $\implies 3x^2 - 5x - 2 = 0 \implies \begin{cases} \boxed{x = -1/3} \implies \boxed{y = 2/3} \\ \boxed{x = 2} \implies \boxed{y = 3} \end{cases}$
- 15 $\begin{cases} x + y = 5 \implies y = 5 - x \\ x^2 - xy + y^2 = 7 \xrightarrow{y=5-x} x^2 - x \cdot (5 - x) + (5 - x)^2 = 7 \end{cases}$
 $\implies x^2 - 5x + x^2 + 25 + x^2 - 10x = 7 \implies 3x^2 - 15x + 18 = 0$
 $\implies \begin{cases} \boxed{x = 2} \implies \boxed{y = 3} \\ \boxed{x = 3} \implies \boxed{y = 2} \end{cases}$
- 16 $\begin{cases} y = 1 + 2x \\ x^2 + y^2 + 6x = 16 \xrightarrow{y=1+2x} x^2 + (1 + 2x)^2 + 6x = 16 \implies x^2 + 1 + 4x^2 + 4x + 6x = 16 \end{cases}$
 $\implies 5x^2 + 10x - 15 = 0 \implies \begin{cases} \boxed{x = 1} \implies \boxed{y = 3} \\ \boxed{x = -3} \implies \boxed{y = -5} \end{cases}$
- 17 $\begin{cases} x = 3y - 1 \\ \frac{1}{x} - \frac{1}{y} = \frac{-1}{2} \xrightarrow{x=3y-1} \frac{1}{3y-1} - \frac{1}{y} = \frac{-1}{2} \implies \frac{2y - 2 \cdot (3y - 1)}{2 \cdot (3y - 1)} = \frac{-y \cdot (3y - 1)}{2 \cdot (3y - 1)} \end{cases}$
 $\implies 2y - 6y + 2 = -3y^2 + y \implies 3y^2 - 5y + 2 = 0$
 $\implies \begin{cases} \boxed{y = 2/3} \implies \boxed{x = 1} \\ \boxed{y = 1} \implies \boxed{x = 2} \end{cases}$
- 18 $\begin{cases} x + y = 6 \implies y = 6 - x \\ xy = 8 \xrightarrow{y=6-x} x \cdot (6 - x) = 8 \implies 6x - x^2 = 8 \implies x^2 - 6x + 8 \end{cases}$
 $\implies \begin{cases} \boxed{x = 2} \implies \boxed{y = 4} \\ \boxed{x = 4} \implies \boxed{y = 2} \end{cases}$
- 19 $\begin{cases} x + y = 6 \implies y = 6 - x \\ xy = 9 \xrightarrow{y=6-x} x \cdot (6 - x) = 9 \implies 6x - x^2 = 9 \implies x^2 - 6x + 9 = 0 \end{cases}$
 $\implies \boxed{x = 3} \implies \boxed{y = 3}$

5.1. Sistemas de ecuaciones no lineales

$$\begin{aligned} 20 \quad & \begin{cases} x - y = 1 \implies x = 1 + y \\ x^2 - y^2 = 17 \xrightarrow{x=1+y} (1+y)^2 - y^2 = 17 \implies 1 + y^2 + 2y - y^2 = 17 \end{cases} \\ & \implies \boxed{y = 8} \implies \boxed{x = 9} \end{aligned}$$

$$21 \quad \begin{cases} x + y = 3 \\ \frac{1}{x} + \frac{1}{y} = \frac{3}{2} \end{cases}$$

$$\begin{aligned} 22 \quad & \begin{cases} 3x - 8y = 5 \implies x = \frac{5+8y}{3} \\ 4xy - 6y = 3 \xrightarrow{\frac{5+8y}{3}} 4 \cdot \frac{5+8y}{3} \cdot y - 6y = 3 \implies \frac{20y + 32y^2}{3} - 6y = 3 \end{cases} \\ & \implies 20y + 32y^2 - 18y = 9 \implies 32y^2 + 2y - 9 = 0 \\ & \implies \begin{cases} \boxed{y = -9/16} \implies \boxed{x = 1/6} \\ \boxed{y = 1/2} \implies \boxed{x = 3} \end{cases} \end{aligned}$$

$$\begin{aligned} 23 \quad & \begin{cases} 3x - 2y = 1 \implies x = \frac{1+2y}{3} \\ 3xy - 4y^2 = 0 \xrightarrow{x=\frac{1+2y}{3}} 3 \cdot \frac{1+2y}{3} \cdot y - 4y^2 = 0 \implies y + 2y^2 - 4y^2 = 0 \end{cases} \\ & \implies 2y^2 - y = 0 \implies y \cdot (2y - 1) = 0 \implies \begin{cases} \boxed{y = 0} \implies \boxed{x = 1/3} \\ \boxed{y = 1/2} \implies \boxed{x = 2/3} \end{cases} \end{aligned}$$

$$\begin{aligned} 24 \quad & \begin{cases} x - y = 5 \implies y = x - 5 \\ 2 \cdot (x + 2y)^2 - (2x + y)^2 = -14 \xrightarrow{y=x-5} 2 \cdot [x + 2 \cdot (x - 5)]^2 - (2x + x - 5)^2 = -14 \end{cases} \\ & \implies 2 \cdot (3x - 10)^2 - (3x - 5)^2 = -14 \\ & \implies 2 \cdot (9x^2 + 100 - 60x) - (9x^2 + 25 - 30x) = -14 \\ & \implies 9x^2 - 90x + 189 = 0 \\ & \implies \begin{cases} \boxed{x = 7} \implies \boxed{y = 2} \\ \boxed{x = 3} \implies \boxed{y = -2} \end{cases} \end{aligned}$$

$$\begin{aligned} 25 \quad & \begin{cases} x + y = 2 \implies y = 2 - x \\ \frac{x - y}{x + y} + \frac{x + y}{x - y} = \frac{5}{2} \xrightarrow{y=2-x} \frac{x - (2 - x)}{x + 2 - x} + \frac{x + 2 - x}{x - (2 - x)} = \frac{5}{2} \implies \frac{2x - 2}{2} + \frac{2}{2x - 2} = \frac{5}{2} \end{cases} \\ & \implies \frac{(2x - 2)^2 + 4}{2 \cdot (2x - 2)} = \frac{5 \cdot (2x - 2)}{2 \cdot (2x - 2)} \\ & \implies 4x^2 + 4 - 8x + 4 = 10x - 10 \implies 4x^2 - 18x + 18 = 0 \\ & \implies \begin{cases} \boxed{x = 3} \implies \boxed{y = -1} \\ \boxed{x = 3/2} \implies \boxed{y = 1/2} \end{cases} \end{aligned}$$

$$\begin{aligned}
26 \quad & \begin{cases} 7x - 2y = 1 \implies y = \frac{7x-1}{2} \\ \frac{x^2 - 3x + 4}{y^2 - 2y + 3} = \frac{1}{3} \xrightarrow{y=\frac{7x-1}{2}} \frac{x^2 - 3x + 4}{\left(\frac{7x-1}{2}\right)^2 - 2 \cdot \frac{7x-1}{2} + 3} = \frac{1}{3} \end{cases} \\
& \implies 3 \cdot (x^2 - 3x + 4) = \frac{49x^2 + 1 - 14x}{4} - 7x + 1 + 3 \\
& \implies 4 \cdot (3x^2 - 2x + 8) = 49x^2 + 1 - 14x \\
& \implies 37x^2 - 6x - 31 = 0 \implies \begin{cases} x = 1 \implies y = 3 \\ x = -31/37 \implies y = -127/37 \end{cases}
\end{aligned}$$

$$\begin{aligned}
27 \quad & \begin{cases} 3 - \frac{2x-1}{x+1} = \frac{y+3}{y+1} \xrightarrow[\circledast]{\circledbullet} 3 - 2 + \frac{3}{x+1} = 1 + \frac{2}{y+1} \implies \frac{3}{x+1} = \frac{2}{y+1} \\ \implies 3y + 3 = 2x + 2 \implies y = \frac{2x-1}{3} \\ 3x + 1 = 2x^2 - y^2 \xrightarrow{y=\frac{2x-1}{3}} 3x + 1 = 2x^2 - \left(\frac{2x-1}{3}\right)^2 \\ \implies 3x + 1 = 2x^2 - \frac{4x^2 + 1 - 4x}{9} \\ \implies 27x + 9 = 18x^2 - 4x^2 - 1 + 4x \implies 14x^2 - 23x - 10 = 0 \\ \implies \begin{cases} x = -5/14 \implies y = -4/7 \\ x = 2 \implies y = 1 \end{cases} \end{cases}
\end{aligned}$$

$$\begin{array}{ccc}
\bullet & \frac{2x-1}{-2x-2} \Big| \frac{x+1}{2} & \quad \quad \quad \circledast \quad \frac{y+3}{-y-1} \Big| \frac{y+1}{1} \\
& -3 & \quad \quad \quad 2
\end{array}$$

$$\begin{aligned}
28 \quad & \begin{cases} xy = 30 \implies y = \frac{30}{x} \\ x^2 + y^2 = 61 \xrightarrow{y=\frac{30}{x}} x^2 + \left(\frac{30}{x}\right)^2 = 61 \implies x^2 + \frac{900}{x^2} = 61 \implies x^4 + 900 = 61x^2 \\ \implies x^4 - 61x^2 + 900 = 0 \xrightarrow{x^2=t} t^2 - 61t + 900 = 0 \\ \implies \begin{cases} t = 25 = x^2 \implies \begin{cases} x = -5 \implies y = -6 \\ x = 5 \implies y = 6 \end{cases} \\ t = 36 = x^2 \implies \begin{cases} x = -6 \implies y = -5 \\ x = 6 \implies y = 5 \end{cases} \end{cases} \end{cases}
\end{aligned}$$

$$\begin{aligned}
29 \quad & \begin{cases} xy + 12 = 0 \implies y = \frac{-12}{x} \\ x^2 + y^2 = 25 \xrightarrow{y=\frac{-12}{x}} x^2 + \left(\frac{-12}{x}\right)^2 = 25 \implies x^2 + \frac{144}{x^2} = 25 \implies x^4 + 144 = 25x^2 \\ \implies x^4 - 25x^2 + 144 = 0 \xrightarrow{x^2=t} t^2 - 25t + 144 = 0 \end{cases}
\end{aligned}$$

5.1. Sistemas de ecuaciones no lineales

$$\Rightarrow \begin{cases} t = 9 = x^2 \Rightarrow \begin{cases} x = -3 \Rightarrow y = 4 \\ x = 3 \Rightarrow y = -4 \end{cases} \\ t = 16 = x^2 \Rightarrow \begin{cases} x = -4 \Rightarrow y = 3 \\ x = 4 \Rightarrow y = -3 \end{cases} \end{cases}$$

$$(30) \begin{cases} xy + 3 = 0 \Rightarrow y = \frac{-3}{x} \\ 2x^2 - 5y^2 = 13 \xrightarrow{y=\frac{-3}{x}} 2x^2 - 5 \cdot \left(\frac{-3}{x}\right)^2 = 13 \Rightarrow 2x^2 - \frac{45}{x^2} = 13 \Rightarrow 2x^4 - 45 = 13x^2 \end{cases}$$

$$\Rightarrow 2x^4 - 13x^2 - 45 = 0 \xrightarrow{x^2=t} 2t^2 - 13t - 45 = 0$$

$$\Rightarrow \begin{cases} t = 9 = x^2 \Rightarrow \begin{cases} x = -3 \Rightarrow y = 1 \\ x = 3 \Rightarrow y = -1 \end{cases} \\ t = -\frac{5}{2} = x^2 \Rightarrow \nexists \text{ Solución} \end{cases}$$

$$(31) \begin{cases} xy = 6 \Rightarrow y = \frac{6}{x} \\ x^2 + xy + y^2 = 19 \xrightarrow[xy=6]{y=\frac{6}{x}} x^2 + 6 + \left(\frac{6}{x}\right)^2 = 19 \Rightarrow x^2 + 6 + \frac{36}{x^2} = 19 \end{cases}$$

$$\Rightarrow x^4 - 13x^2 + 36 = 0 \xrightarrow{x^2=t} t^2 - 13t + 36 = 0$$

$$\Rightarrow \begin{cases} t = 9 = x^2 \Rightarrow \begin{cases} x = -3 \Rightarrow y = -2 \\ x = 3 \Rightarrow y = 2 \end{cases} \\ t = 4 = x^2 \Rightarrow \begin{cases} x = -2 \Rightarrow y = -3 \\ x = 2 \Rightarrow y = 3 \end{cases} \end{cases}$$

$$(32) \begin{cases} x^2 - yx + y^2 = 7 \xrightarrow{E_2-E_1} xy = 6 \Rightarrow y = \frac{6}{x} \\ x^2 + y^2 = 13 \xrightarrow{y=\frac{6}{x}} x^2 + \left(\frac{6}{x}\right)^2 = 13 \Rightarrow x^2 + \frac{36}{x^2} = 13 \Rightarrow x^4 + 36 = 13x^2 \end{cases}$$

$$\xrightarrow{x^2=t} t^2 - 13t + 36 = 0 \Rightarrow \begin{cases} t = 9 = x^2 \Rightarrow \begin{cases} x = -3 \Rightarrow y = -2 \\ x = 3 \Rightarrow y = 2 \end{cases} \\ t = 4 = x^2 \Rightarrow \begin{cases} x = -2 \Rightarrow y = -3 \\ x = 2 \Rightarrow y = 3 \end{cases} \end{cases}$$

$$(33) \begin{cases} x^2 + 2y^2 = 22 \Rightarrow x^2 = 22 - 2y^2 \Rightarrow x = \pm\sqrt{22 - 2y^2} \\ 2x^2 - y^2 = -1 \xrightarrow{x^2=22-2y^2} 2 \cdot (22 - 2y^2) - y^2 = -1 \Rightarrow 44 - 4y^2 - y^2 = -1 \end{cases}$$

$$\Rightarrow y^2 = 9 \Rightarrow \begin{cases} y = -3 \xrightarrow{x=\pm\sqrt{22-2y^2}} \begin{cases} x = -2 \\ x = 2 \end{cases} \\ y = 3 \xrightarrow{x=\pm\sqrt{22-2y^2}} \begin{cases} x = -2 \\ x = 2 \end{cases} \end{cases}$$

$$(34) \begin{cases} 2x^2 - 3y^2 = -6 \\ 4x^2 - y^2 = 8 \end{cases} \xrightarrow{E_2 - 2E_1} 5y^2 = 20 \Rightarrow \begin{cases} y = -2 \xrightarrow{x=\pm\sqrt{\frac{3y^2-6}{2}}} \begin{cases} x = -\sqrt{3} \\ x = \sqrt{3} \end{cases} \\ y = 2 \xrightarrow{x=\pm\sqrt{\frac{3y^2-6}{2}}} \begin{cases} x = -\sqrt{3} \\ x = \sqrt{3} \end{cases} \end{cases}$$

$$(35) \begin{cases} y^2 = 16 + 4x \\ x^2 + y^2 + 9x + 14 = 0 \end{cases} \xrightarrow{y^2=16+4x} x^2 + 16 + 4x + 9x + 14 = 0 \Rightarrow x^2 + 13x + 30 = 0$$

$$\Rightarrow \begin{cases} x = -3 \xrightarrow{y^2=16+4x} \begin{cases} y = -2 \\ y = 2 \end{cases} \\ x = -10 \xrightarrow{y^2=16+4x} \nexists \text{ Solución} \end{cases}$$

$$(36) \begin{cases} y^2 = 6x \\ x^2 - y^2 + 8 = 0 \end{cases} \xrightarrow{y^2=6x} x^2 - 6x + 8 = 0$$

$$\Rightarrow \begin{cases} x = 2 \Rightarrow \begin{cases} y = -2\sqrt{3} \\ y = 2\sqrt{3} \end{cases} \\ x = 4 \Rightarrow \begin{cases} y = -2\sqrt{6} \\ y = 2\sqrt{6} \end{cases} \end{cases}$$

$$(37) \begin{cases} x^2 + xy = 77 \Rightarrow y = \frac{77 - x^2}{x} \\ xy + y^2 = 44 \xrightarrow{y=\frac{77-x^2}{x}} x \cdot \frac{77 - x^2}{x} + \left(\frac{77 - x^2}{x}\right)^2 = 44 \end{cases}$$

$$\Rightarrow 77 - x^2 + \frac{5929 + x^4 - 154x^2}{x^2} = 44 \Rightarrow 121x^2 - 5929 = 0$$

$$\Rightarrow \begin{cases} x = -7 \Rightarrow y = -4 \\ x = 7 \Rightarrow y = 4 \end{cases}$$

$$(38) \begin{cases} y^2 + xy = 45 \Rightarrow x = \frac{45 - y^2}{y} \\ x^2 + 2xy = 56 \xrightarrow{x=\frac{45-y^2}{y}} \left(\frac{45 - y^2}{y}\right)^2 + 2y \cdot \frac{45 - y^2}{y} = 56 \end{cases}$$

5.1. Sistemas de ecuaciones no lineales

$$\Rightarrow \frac{2025 + y^4 - 90y^2}{y^2} + 90 - 2y^2 = 56 \Rightarrow y^4 + 56y^2 - 2025 = 0$$

$$\xRightarrow{y^2=t} t^2 + 56t - 2025 = 0$$

$$\Rightarrow \begin{cases} t = -81 = y^2 \Rightarrow \nexists \text{ Solución} \\ t = 25 = y^2 \Rightarrow \begin{cases} y = -5 \Rightarrow x = -4 \\ y = 5 \Rightarrow x = 4 \end{cases} \end{cases}$$

$$\begin{aligned} (39) \quad & \begin{cases} xy = 6 \Rightarrow y = \frac{6}{x} \\ \frac{1}{x} + \frac{1}{y} = \frac{5}{6} \xRightarrow{y=\frac{6}{x}} \frac{1}{x} + \frac{x}{6} = \frac{5}{6} \Rightarrow \frac{6+x^2}{6x} = \frac{5x}{6x} \Rightarrow 6+x^2 = 5x \Rightarrow x^2 - 5x + 6 = 0 \end{cases} \\ & \Rightarrow \begin{cases} x = 2 \Rightarrow y = 3 \\ x = 3 \Rightarrow y = 2 \end{cases} \end{aligned}$$

$$\begin{aligned} (40) \quad & \begin{cases} \frac{1}{x} - \frac{1}{y} = 1 \Rightarrow \frac{1}{y} = \frac{1}{x} - 1 \Rightarrow \frac{1}{x} = \frac{1-x}{x} \\ \frac{1}{x^2} + \frac{1}{y^2} = 13 \xRightarrow{\frac{1}{x}=\frac{1-x}{x}} \frac{1}{x^2} + \left(\frac{1-x}{x}\right)^2 = 13 \Rightarrow \frac{1}{x^2} + \frac{1+x^2-2x}{x^2} = 13 \end{cases} \\ & \Rightarrow \frac{1+1+x^2-2x}{x^2} = \frac{13x^2}{x^2} \Rightarrow 12x^2 + 2x - 2 = 0 \\ & \Rightarrow \begin{cases} x = -1/2 \Rightarrow y = -1/3 \\ x = 1/3 \Rightarrow y = 1/2 \end{cases} \end{aligned}$$

$$\begin{aligned} (41) \quad & \begin{cases} x + \frac{2}{y} = 1 \Rightarrow x = 1 - \frac{2}{y} = \frac{y-2}{y} \Rightarrow \frac{1}{x} = \frac{y}{y-2} \\ y + \frac{1}{x} = 6 \xRightarrow{\frac{1}{x}=\frac{y}{y-2}} y + \frac{y}{y-2} = 6 \Rightarrow \frac{y \cdot (y-2) + y}{y-2} = \frac{6 \cdot (y-2)}{y-2} \end{cases} \\ & \Rightarrow y^2 - 2y + y = 6y - 12 \Rightarrow y^2 - 7y + 12 = 0 \\ & \Rightarrow \begin{cases} y = 3 \Rightarrow x = 1/3 \\ y = 4 \Rightarrow x = 1/2 \end{cases} \end{aligned}$$

$$\begin{aligned} (42) \quad & \begin{cases} y + \frac{x}{y} = \frac{21}{2} \\ x - \frac{x}{y} = \frac{9}{2} \end{cases} \xRightarrow{E_1+E_2} y + x = 15 \Rightarrow y = 15 - x \\ & \Rightarrow x - \frac{x}{y} = \frac{9}{2} \xRightarrow{y=15-x} x - \frac{x}{15-x} = \frac{9}{2} \Rightarrow \frac{x \cdot (15-x) - x}{15-x} = \frac{9}{2} \\ & \Rightarrow \frac{14x - x^2}{15-x} = \frac{9}{2} \Rightarrow 28x - 2x^2 = 135 - 9x \Rightarrow 2x^2 - 37x + 135 = 0 \end{aligned}$$

$$\Rightarrow \begin{cases} \boxed{x = 5} \Rightarrow \boxed{y = 10} \\ \boxed{x = 27/2} \Rightarrow \boxed{y = 3/2} \end{cases}$$

$$(43) \begin{cases} x^2 + y^2 = 25 + 2xy \\ x^2 + 2xy + y^2 = 169 \end{cases} \xrightarrow{E_1 - E_2} -2xy = -144 + 2xy \Rightarrow 4xy = 144 \Rightarrow y = \frac{36}{x}$$

$$\Rightarrow x^2 + 2xy + y^2 = 169 \xrightarrow{y = \frac{36}{x}} x^2 + 2x \cdot \frac{36}{x} + \left(\frac{36}{x}\right)^2 = 169 \Rightarrow x^2 + 72 + \frac{1296}{x^2} = 169$$

$$\Rightarrow x^4 + 72x^2 + 1296 = 169x^2 \Rightarrow x^4 - 97x^2 + 1296 = 0 \xrightarrow{x^2 = t} t^2 - 97t + 1296 = 0$$

$$\Rightarrow \begin{cases} t = 16 = x^2 \Rightarrow \begin{cases} \boxed{x = -4} \Rightarrow \boxed{y = -9} \\ \boxed{x = 4} \Rightarrow \boxed{y = 9} \end{cases} \\ t = 81 = x^2 \Rightarrow \begin{cases} \boxed{x = -9} \Rightarrow \boxed{y = -4} \\ \boxed{x = 9} \Rightarrow \boxed{y = 4} \end{cases} \end{cases}$$

$$(44) \begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{5}{6} \\ \frac{1}{x} - \frac{1}{y} = \frac{1}{6} \end{cases} \xrightarrow{E_1 + E_2} \frac{2}{x} = 1 \Rightarrow \boxed{x = 2} \xrightarrow{\frac{1}{2} + \frac{1}{y} = \frac{5}{6}} \frac{1}{y} = \frac{1}{3} \Rightarrow \boxed{y = 3}$$

$$(45) \begin{cases} \frac{2}{x} + \frac{3}{y} = \frac{17}{12} \\ \frac{1}{x} - \frac{2}{y} = \frac{-1}{6} \end{cases} \xrightarrow{E_1 - 2E_2} \frac{7}{y} = \frac{7}{4} \Rightarrow \boxed{y = 4} \xrightarrow{\frac{1}{x} - \frac{2}{4} = \frac{-1}{6}} \frac{1}{x} = \frac{1}{3} \Rightarrow \boxed{x = 3}$$

$$(46) \begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{9}{20} \\ x^2y + xy^2 = 180 \end{cases}$$

$$(47) \begin{cases} x - y = 105 \Rightarrow y = x - 105 \\ \sqrt{x} + \sqrt{y} = 15 \xrightarrow{y = x - 105} \sqrt{x} + \sqrt{x - 105} = 15 \Rightarrow \sqrt{x - 105} = 15 - \sqrt{x} \end{cases}$$

$$\Rightarrow x - 105 = 225 + x - 30\sqrt{x} \Rightarrow 11 = \sqrt{x}$$

$$\Rightarrow \boxed{x = 121} \Rightarrow \boxed{y = 16}$$

$$(48) \begin{cases} x + y = 5\sqrt{y} \Rightarrow x = 5\sqrt{y} - y \\ \sqrt{x} - \sqrt{y} = 1 \xrightarrow{x = 5\sqrt{y} - y} \sqrt{5\sqrt{y} - y} - \sqrt{y} = 1 \Rightarrow \sqrt{5\sqrt{y} - y} = 1 + \sqrt{y} \end{cases}$$

$$\Rightarrow 5\sqrt{y} - y = 1 + y + 2\sqrt{y} \Rightarrow 3\sqrt{y} = 1 + 2y$$

$$\Rightarrow 9y = 1 + 4y^2 + 4y \Rightarrow 4y^2 - 5y + 1 = 0$$

$$\Rightarrow \begin{cases} \boxed{y = 1/4} \Rightarrow \boxed{x = 9/4} \\ \boxed{y = 1} \Rightarrow \boxed{x = 5} \end{cases}$$

6.1. Ecuaciones exponenciales

① $2^x = 128$

② $3^{-x} = 27$

③ $5 \cdot 2^x = \frac{5}{4}$

④ $2^{x-1} = \frac{1}{16}$

⑤ $3^{x-9} = 27$

⑥ $\frac{1}{5^x} = 25$

⑦ $0,1^x = \frac{1}{100}$

⑧ $0,5^x = 16$

⑨ $2^{5x+3} = 1000$

⑩ $10^{3x+5} = 3584$

METER AQUI LOS EXPONENCIA-
LES FÁCILES

⑪ $4^x = \frac{1}{64}$

⑫ $2^{x-5} = 32$

⑬ $3^{6-x} = 27^{x-2}$

⑭ $32^{x-2} = 2$

⑮ $2^{x+1} = 8$

⑯ $2^{x+1} = 16$

⑰ $2^{x+1} = 128$

⑱ $16^{2x-4} = 1$

⑲ $3^{x+1} + 3^{x-2} + 3^x + 3^{x-1} = 120$

⑳ $4^x + 2^{x-1} = \frac{1}{2}$

㉑ $2^{-x+5} = 8^{x+3}$

㉒ $2^x + 2^{x-1} + 2^{x+1} + 2^{x-3} = 29$

㉓ $4^{x+1} - 3 \cdot 2^{x+2} - 16 = 0$

㉔ $2^{x+1} + 4^{x-1} = 96$

㉕ $4^{x-2} + 2^{x+1} = 20$

㉖ $(3^x)^3 \cdot 3^{-x} = \frac{3^x}{3^{x^2}}$

㉗ $2^{x^2+2x-5} \cdot 4^{2-x} = 8^{x+1}$

㉘ $3^{5/3} \cdot \frac{\sqrt{3^x}}{9^x} = \frac{1}{81}$

㉙ $4^3 \cdot (4^{x-1} + 4^x - 4^{x+1}) = -11$

㉚ $\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}} = 2^x$

㉛ $2^x \cdot 3^{x-1} = 2592$

㉜ $4 \cdot \left(2^x + \frac{1}{2^{x-1}}\right) = 33$

6.1. Ecuaciones exponenciales

$$33 \quad \frac{2}{4^x - 2^x} = 1$$

$$34 \quad 8^x - 3 \cdot 4^x - 3 \cdot 2^{x+1} + 8 = 0$$

$$35 \quad 2^{3-x} + 2^{1+x} = 10$$

$$36 \quad 2^{x+1} + 2^{x-1} = \frac{5}{2}$$

$$37 \quad 3^{x-1} - 3^{x-2} = 6$$

$$38 \quad 3^{2x} - 3^x - 6 = 0$$

$$39 \quad 3^x + 3^{x+1} = 4$$

$$40 \quad 3^{2x+3} = 2187$$

$$41 \quad 3^{\frac{x+1}{x-2}} = \frac{1}{9}$$

$$42 \quad 3^{x^2-3x+3} = 3$$

$$43 \quad 10^{\frac{x^2-1}{x+1}} = 10$$

$$44 \quad 3^{2x-1} - 3^{x+1} = 0$$

$$45 \quad 5^{2x+1} + 3 \cdot 5^{6x-3} = 500$$

$$46 \quad 4^{x-2} - 2^{x+1} = -12$$

$$47 \quad 3^{2 \cdot (x+2)} - 4 \cdot 3^x - 77 = 0$$

$$48 \quad 3^{2x+5} = 27^{x+2}$$

$$49 \quad 2^{2x} - 5 \cdot 2^x + 4 = 0$$

$$50 \quad 3^{1+x} + \left(\frac{1}{3}\right)^{-x} = 36$$

$$51 \quad 2^{1-x} - \left(\frac{1}{2}\right)^x = 4$$

$$52 \quad 2^x + 2^{1-x} = 3$$

$$53 \quad 3^x + \frac{1}{3^{1+x}} = \frac{28}{9}$$

$$54 \quad 4^{7x-1} = 8000$$

$$55 \quad 3^{2x+1} - 12 \cdot 3^x + 3^2 = 0$$

$$56 \quad 3^{x+1} - 3^x - 2 \cdot 3^{x-1} = 12$$

$$57 \quad 5^{x^2-3x} = 625$$

$$58 \quad 2^{x^2+3} = 200$$

$$59 \quad 3^{x^2+x+1} = 2187$$

$$60 \quad 3^{-x} + 3^{x+1} = 4$$

$$61 \quad 3^{2x+3} = 2187$$

$$62 \quad 2^{3x} - 2^{2x} - 4 = 0$$

$$63 \quad 3^{x-1} + 3^x - 3^{x+1} = -45$$

$$64 \quad 25^{x+2} = 5^{-x-2}$$

$$65 \quad 4^{x-1} = 2^{x+1}$$

$$66 \quad 3^{x+2} = 9$$

$$67 \quad 2^{x+1} = 4^x$$

$$68 \quad 3^{x+1} + 3^{x-2} + 3^x + 3^{x-1} = 120$$

$$69 \quad \frac{1}{4^x - 2^x} = -1$$

$$70 \quad 8^x - 3 \cdot 4^x - 2^{x+2} + 8 = 0$$

$$71 \quad 2^{2x} - 3 \cdot 2^x - 4 = 0$$

$$72 \quad 3^x + 3^{x-1} - 3^{x-2} = 11$$

$$73 \quad 2^x + 2^{-x} = \frac{65}{8}$$

$$74 \quad 2^{2x+1} = 8^{x-1}$$

$$75 \quad 3^{x-1} = 3^{x^2-1}$$

$$76 \quad \frac{2^{3x+1}}{2^{x^2}} = \frac{4^{x-1}}{2^x}$$

$$77 \quad 3 \cdot 27^{x-2} = 9^x$$

$$78 \quad 5^{x+4} = 125^{x-4}$$

$$79 \quad \frac{1}{81^{6-x}} = 3^{4-x}$$

$$80 \quad 32^{2x-3} = 2^{x+3}$$

$$\textcircled{81} \quad 125^{x+2} = 5^{2x}$$

$$\textcircled{82} \quad 256^x = 4 \cdot 4^{2x-3}$$

$$\textcircled{83} \quad 3^{x^3 - \frac{x^2}{10} - \frac{3}{10}} = 1$$

$$\textcircled{84} \quad \frac{2^{x^3 - x^2 - 5x}}{8} = 1$$

$$\textcircled{85} \quad 3^{3x} + 5 \cdot 3^{2x-1} - 11 \cdot 3^{x-1} + 1 = 0$$

$$\textcircled{86} \quad 4 \cdot \left(\frac{1}{2}\right)^{3x} = 9 \cdot 2^{-2x} - 2^{-x+1}$$

$$\textcircled{87} \quad \left(\frac{3}{2}\right)^{3x+1} + 1 = \frac{3^{2x}}{2} + \left(\frac{3}{2}\right)^{x+1}$$

HTTPS://APRENDECONMIGOMELON.COM

6.1. Ecuaciones exponenciales

$$\textcircled{1} \quad 2^x = 128 \Rightarrow 2^x = 2^7 \Rightarrow \boxed{x = 7}$$

$$\textcircled{2} \quad 3^{-x} = 27 \Rightarrow 3^{-x} = 3^3 \Rightarrow -x = 3 \Rightarrow \boxed{x = -3}$$

$$\textcircled{3} \quad 5 \cdot 2^x = \frac{5}{4} \Rightarrow 2^x = \frac{1}{4} \Rightarrow 2^x = 2^{-2} \Rightarrow \boxed{x = -2}$$

$$\textcircled{4} \quad 2^{x-1} = \frac{1}{16} \Rightarrow 2^{x-1} = 2^{-4} \Rightarrow x-1 = -4 \Rightarrow \boxed{x = -3}$$

$$\textcircled{5} \quad 3^{x-9} = 27 \Rightarrow 3^{x-9} = 3^3 \Rightarrow x-9 = 3 \Rightarrow \boxed{x = 12}$$

$$\textcircled{6} \quad \frac{1}{5^x} = 25 \Rightarrow 5^{-x} = 5^2 \Rightarrow -x = 2 \Rightarrow \boxed{x = -2}$$

$$\textcircled{7} \quad 0,1^x = \frac{1}{100} \Rightarrow \left(\frac{1}{10}\right)^x = \frac{1}{10} \Rightarrow 10^{-x} = 10^{-1} \Rightarrow -x = -1 \Rightarrow \boxed{x = 1}$$

$$\textcircled{8} \quad 0,5^x = 16 \Rightarrow \left(\frac{5}{10}\right)^x = 2^4 \Rightarrow \left(\frac{1}{2}\right)^x = 2^4 \Rightarrow 2^{-x} = 2^4 \Rightarrow \boxed{x = -4}$$

$$\begin{aligned} \textcircled{9} \quad 2^{5x+3} &= 1000 \Rightarrow 2^{5x} \cdot 2^3 = 1000 \Rightarrow 8 \cdot 2^{5x} = 1000 \Rightarrow 2^{5x} = 125 \\ &\Rightarrow \log_2 2^{5x} = \log_2 125 \Rightarrow 5x \cdot \log_2 2 = \log_2 125 \Rightarrow 5x = \log_2 125 \\ &\Rightarrow x = \frac{1}{5} \cdot \log_2 125 \Rightarrow \boxed{x = \log_2 \sqrt[5]{125}} \end{aligned}$$

$$\begin{aligned} \textcircled{10} \quad 10^{3x+5} &= 3584 \Rightarrow \log 10^{3x+5} = \log 3584 \Rightarrow (3x+5) \cdot \log 10 = \log 3584 \\ &\Rightarrow \boxed{x = \frac{\log 3584 - 5}{3}} \end{aligned}$$

$$\textcircled{11} \quad 4^x = \frac{1}{64} \Rightarrow 4^x = \frac{1}{2^6} \Rightarrow 4^x = \frac{1}{4^3} \Rightarrow 4^x = 4^{-3} \Rightarrow \boxed{x = -3}$$

$$\textcircled{12} \quad 2^{x-5} = 32 \Rightarrow 2^{x-5} = 2^5 \Rightarrow x-5 = 5 \Rightarrow \boxed{x = 10}$$

$$\textcircled{13} \quad 3^{6-x} = 27^{x-2} \Rightarrow 3^{6-x} = (3^3)^{x-2} \Rightarrow 3^{6-x} = 3^{3x-6} \Rightarrow 6-x = 3x-6 \Rightarrow \boxed{x = 3}$$

$$\textcircled{14} \quad 32^{x-2} = 2 \Rightarrow (2^5)^{x-2} = 2 \Rightarrow 2^{5x-10} = 2 \Rightarrow 5x-10 = 1 \Rightarrow \boxed{x = \frac{11}{5}}$$

$$\textcircled{15} \quad 2^{x+1} = 8 \Rightarrow 2^{x+1} = 2^3 \Rightarrow x+1 = 3 \Rightarrow \boxed{x = 2}$$

$$\textcircled{16} \quad 2^{x+1} = 16 \Rightarrow 2^{x+1} = 2^4 \Rightarrow x+1 = 4 \Rightarrow \boxed{x = 3}$$

$$\textcircled{17} \quad 2^{x+1} = 128 \Rightarrow 2^{x+1} = 2^7 \Rightarrow x+1 = 7 \Rightarrow \boxed{x = 6}$$

$$\textcircled{18} \quad 16^{2x-4} = 1 \Rightarrow 16^{2x-4} = 16^0 \Rightarrow 2x-4 = 0 \Rightarrow \boxed{x = 2}$$

$$\begin{aligned} \textcircled{19} \quad 3^{x+1} + 3^{x-2} + 3^x + 3^{x-1} &= 120 \Rightarrow 3 \cdot 3^x + \frac{3^x}{3^2} + 3^x + \frac{3^x}{3} = 120 \\ &\Rightarrow 27 \cdot 3^x + 3^x + 9 \cdot 3^x + 3 \cdot 3^x = 1080 \Rightarrow 40 \cdot 3^x = 1080 \Rightarrow 3^x = 27 \\ &\Rightarrow 3^x = 3^3 \Rightarrow \boxed{x = 3} \end{aligned}$$

$$(20) \quad 4^x + 2^{x-1} = \frac{1}{2} \implies 2^{2x} + \frac{2^x}{2} = \frac{1}{2} \xrightarrow[2^{2x}=t^2]{2^x=t} 2t^2 + t = 1 \implies 2t^2 + t - 1 = 0$$

$$\implies \begin{cases} t = -1 = 2^x \implies \nexists \text{ Solución} \\ t = \frac{1}{2} = 2^{-1} = 2^x \implies \boxed{x = -1} \end{cases}$$

$$(21) \quad 2^{-x+5} = 8^{x+3} \implies \frac{2^5}{2^x} = 2^{3 \cdot (x+3)} \implies \frac{2^5}{2^x} = 2^{3x+9} \implies \frac{2^5}{2^x} = 2^9 \cdot 2^{3x}$$

$$\implies \frac{2^5}{2^9} = 2^x \cdot 2^{3x} \implies 2^{-4} = 2^{4x} \implies 4x = -4 \implies \boxed{x = -1}$$

$$(22) \quad 2^x + 2^{x-1} + 2^{x+1} + 2^{x-3} = 29 \implies 2^x + \frac{2^x}{2} + 2 \cdot 2^x + \frac{2^x}{2^3} = 29 \xrightarrow[2^{2x}=t^2]{2^x=t} 2t^2 + t = 1$$

$$\implies t + \frac{t}{2} + 2t + \frac{t}{8} = 29 \implies 8t + 4t + 16t + t = 232 \implies 29t = 232$$

$$\implies t = 8 = 2^3 = 2^x \implies \boxed{t = 3}$$

$$(23) \quad 4^{x+1} - 3 \cdot 2^{x+2} - 16 = 0 \implies 2^{2 \cdot (x+1)} - 3 \cdot 2^2 \cdot 2^x - 16 = 0$$

$$\implies 2^{2x+2} - 12 \cdot 2^x - 16 = 0 \implies 2^2 \cdot 2^{2x} - 12 \cdot 2^x - 16 = 0 \xrightarrow[2^{2x}=t^2]{2^x=t} 4t^2 - 12t - 16 = 0$$

$$\implies \begin{cases} t = -1 = 2^x \implies \nexists \text{ Solución} \\ t = 4 = 2^2 = 2^x \implies \boxed{x = 2} \end{cases}$$

$$(24) \quad 2^{x+1} + 4^{x-1} = 96 \implies 2 \cdot 2^x + 2^{2 \cdot (x-1)} = 96 \implies 2 \cdot 2^x + 2^{2x-2} = 96$$

$$\implies 2 \cdot 2^x + \frac{2^{2x}}{2^2} = 96 \xrightarrow[2^{2x}=t^2]{2^x=t} 2t + \frac{t^2}{4} = 96 \implies 8t + t^2 = 384 \implies t^2 + 8t - 384 = 0$$

$$\implies \begin{cases} t = -24 = 2^x \implies \nexists \text{ Solución} \\ t = 16 = 2^4 = 2^x \implies \boxed{x = 4} \end{cases}$$

$$(25) \quad 4^{x-2} + 2^{x+1} = 20 \implies 2^{2 \cdot (x-2)} + 2 \cdot 2^x = 20 \implies 2^{2x-4} + 2 \cdot 2^x = 20 \implies \frac{2^{2x}}{2^4} + 2 \cdot 2^x = 20$$

$$\xrightarrow[2^{2x}=t^2]{2^x=t} \frac{t^2}{16} + 2t - 20 = 0 \implies t^2 + 32t - 320 = 0 \implies \begin{cases} t = -40 = 2^x \implies \nexists \text{ Solución} \\ t = 8 = 2^3 = 2^x \implies \boxed{x = 3} \end{cases}$$

$$(26) \quad (3^x)^3 \cdot 3^{-x} = \frac{3^x}{3^{x^2}} \implies 3^{3x-x} = 3^{x-x^2} \implies 2x = x - x^2 \implies x^2 + x = 0$$

$$\implies x \cdot (x + 1) = 0 \begin{cases} \boxed{x = 0} \\ \boxed{x = -1} \end{cases}$$

$$(27) \quad 2^{x^2+2x-5} \cdot 4^{2-x} = 8^{x+1} \implies 2^{x^2+2x-5} \cdot 2^{2 \cdot (2-x)} = 2^{3 \cdot (x+1)} \implies 2^{x^2+2x-5} \cdot 2^{4-2x} = 2^{3x+3}$$

$$\implies 2^{x^2-1} = 2^{3x+3} \implies x^2 - 1 = 3x + 3 \implies x^2 - 3x - 4 = 0 \implies \begin{cases} \boxed{x = -1} \\ \boxed{x = 4} \end{cases}$$

$$(28) \quad 3^{5/3} \cdot \frac{\sqrt{3^x}}{9^x} = \frac{1}{81} \implies 3^{5/3} \cdot \frac{3^{x/2}}{3^{2x}} = \frac{1}{3^4} \implies 3^{\frac{5}{3} + \frac{x}{2} - 2x} = 3^{-4} \implies \frac{5}{3} + \frac{x}{2} - 2x = -4$$

6.1. Ecuaciones exponenciales

$$\Rightarrow \frac{10 + 3x - 12x}{6} = -\frac{24}{6} \Rightarrow 10 - 9x = -24 \Rightarrow \boxed{x = \frac{34}{9}}$$

$$\begin{aligned} (29) \quad 4^3 \cdot (4^{x-1} + 4^x - 4^{x+1}) &= -11 \Rightarrow 4^3 \cdot \left(\frac{4^x}{4} + 4^x - 4 \cdot 4^x \right) = -11 \\ \Rightarrow 4^{\cancel{3}2} \cdot \frac{4^x + 4 \cdot 4^x - 16 \cdot 4^x}{\cancel{4}} &= -11 \Rightarrow 4^2 \cdot (\cancel{11} \cdot 4^x) = \cancel{11} \\ \Rightarrow 4^x &= 4^{-2} \Rightarrow \boxed{x = -2} \end{aligned}$$

$$(30) \quad \sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}} = 2^x \Rightarrow \sqrt[16]{2^{15}} = 2^x \Rightarrow 2^{15/16} = 2^x \Rightarrow \boxed{x = \frac{15}{16}}$$

$$\begin{aligned} (31) \quad 2^x \cdot 3^{x-1} &= 2592 \Rightarrow 2^x \cdot \frac{3^3}{3} = 2592 \Rightarrow 2^x \cdot 3^x = 7776 \Rightarrow (2 \cdot 3)^x = 2^5 \cdot 3^5 \\ \Rightarrow 6^x &= (2 \cdot 3)^5 \Rightarrow 6^x = 6^5 \Rightarrow \boxed{x = 5} \end{aligned}$$

$$\begin{aligned} (32) \quad 4 \cdot \left(2^x + \frac{1}{2^{x-1}} \right) &= 33 \Rightarrow 4 \cdot \left(2^x + \frac{2}{2^x} \right) = 33 \Rightarrow 4 \cdot (2^{2x} + 2) = 33 \cdot 2^x \\ \Rightarrow 4 \cdot 2^{2x} + 8 &= 33 \cdot 2^x \xrightarrow[2^{2x}=t^2]{2^x=t} 4t^2 - 33t + 8 = 0 \Rightarrow \begin{cases} t = \frac{1}{4} = 2^2 = 2^x \Rightarrow \boxed{x = -2} \\ t = 8 = 2^3 = 2^x \Rightarrow \boxed{x = 3} \end{cases} \end{aligned}$$

$$\begin{aligned} (33) \quad \frac{2}{4^x - 2^x} &= 1 \Rightarrow \frac{2}{2^{2x} - 2^x} = 1 \xrightarrow[2^{2x}=t^2]{2^x=t} \frac{2}{t^2 - t} = 1 \Rightarrow t^2 - t = 2 \Rightarrow t^2 - t - 2 = 0 \\ \Rightarrow \begin{cases} t = -1 = 2^x \Rightarrow \nexists \text{ Solución} \\ t = 2 = 2^x \Rightarrow \boxed{x = 1} \end{cases} \end{aligned}$$

$$\begin{aligned} (34) \quad 8^x - 3 \cdot 4^x - 3 \cdot 2^{x+1} + 8 &= 0 \Rightarrow 2^{3x} - 3 \cdot 2^{2x} - 3 \cdot 2 \cdot 2^x + 8 = 0 \xrightarrow[2^{2x}=t^2]{2^x=t} t^3 - 3t^2 - 6t + 8 = 0 \\ \Rightarrow (t-1) \cdot (t^2 - 2t - 8) &\Rightarrow \begin{cases} t = -2 = 2^x \Rightarrow \nexists \text{ Sol.} \\ t = 1 = 2^x \Rightarrow \boxed{x = 0} \\ t = 4 = 2^2 = 2^x \Rightarrow \boxed{x = 2} \end{cases} \end{aligned}$$

$\odot \begin{array}{c|ccc|c} 1 & 1 & -3 & -6 & 8 \\ & & 1 & -2 & -8 \\ \hline & 1 & -2 & -8 & 0 \end{array}$

$$\begin{aligned} (35) \quad 2^{3-x} + 2^{1+x} &= 10 \Rightarrow \frac{2^3}{2^x} + 2 \cdot 2^x = 10 \Rightarrow 8 + 2 \cdot 2^{2x} = 10 \cdot 2^x \xrightarrow[2^{2x}=t^2]{2^x=t} 8 + 2t^2 = 10t \\ \Rightarrow t^2 - 5t + 4 &= 0 \Rightarrow \begin{cases} t = 1 = 2^x \Rightarrow \boxed{x = 0} \\ t = 4 = 2^2 = 2^x \Rightarrow \boxed{x = 2} \end{cases} \end{aligned}$$

$$\begin{aligned} (36) \quad 2^{x+1} + 2^{x-1} &= \frac{5}{2} \Rightarrow 2 \cdot 2^x + \frac{2^x}{2} = \frac{5}{2} \Rightarrow 4 \cdot 2^x + 2^x = 10 \Rightarrow 5 \cdot 2^x = 10 \\ \Rightarrow 2^x &= 2 \Rightarrow \boxed{x = 2} \end{aligned}$$

$$\begin{aligned} (37) \quad 3^{x-1} - 3^{x-2} &= 6 \Rightarrow \frac{3^x}{3} - \frac{3^x}{3^2} = 6 \Rightarrow 3 \cdot 3^x - 3^x = 54 \Rightarrow 2 \cdot 3^x = 54 \Rightarrow 3^x = 27 \\ \Rightarrow 3^x &= 3^3 \Rightarrow \boxed{x = 3} \end{aligned}$$

$$(38) \quad 3^{2x} - 3^x - 6 = 0 \xrightarrow[3^{2x}=t^2]{3^x=t} \Rightarrow t^2 - t - 6 = 0 \Rightarrow \begin{cases} t = 3 = 3^x \Rightarrow \boxed{x = 1} \\ t = -2 = 3^x \Rightarrow \nexists \text{ Solución} \end{cases}$$

$$(39) \quad 3^x + 3^{x+1} = 4 \Rightarrow 3^x + 3 \cdot 3^x = 4 \Rightarrow 4 \cdot 3^x = 4 \Rightarrow 3^x = 1 \Rightarrow \boxed{x = 0}$$

$$(40) \quad 3^{2x+3} = 2187 \Rightarrow 3^{2x+3} = 3^7 \Rightarrow 2x + 3 = 7 \Rightarrow \boxed{x = 2}$$

$$(41) \quad 3^{\frac{x+1}{x-2}} = \frac{1}{9} \Rightarrow 3^{\frac{x+1}{x-2}} = 3^{-2} \Rightarrow \frac{x+1}{x-2} = -2 \Rightarrow x+1 = -2x+4 \\ \Rightarrow 3x = 3 \Rightarrow \boxed{x = 1}$$

$$(42) \quad 3^{x^2-3x+3} = 3 \Rightarrow x^2 - 3x + 3 = 1 \Rightarrow x^2 - 3x + 2 = 0 \Rightarrow \begin{cases} \boxed{x = 1} \\ \boxed{x = 2} \end{cases}$$

$$(43) \quad 10^{\frac{x^2-1}{x+1}} = 10 \Rightarrow \frac{x^2-1}{x+1} = 1 \Rightarrow x^2 - 1 = x + 1 \Rightarrow x^2 - x - 2 = 0 \\ \Rightarrow \begin{cases} x = -1 \\ \boxed{x = 2} \end{cases}$$

$$(44) \quad 3^{2x-1} - 3^{x+1} = 0 \Rightarrow 3^{2x-1} = 3^{x+1} \Rightarrow 2x - 1 = x + 1 \Rightarrow \boxed{x = 2}$$

$$(45) \quad 5^{2x+1} + 3 \cdot 5^{6x-3} = 500 \Rightarrow 5 \cdot 5^{2x} + \frac{3 \cdot 5^{6x}}{5^3} = 500 \Rightarrow 5^4 \cdot 5^{2x} + 3 \cdot 5^{6x} = 500 \cdot 5^3 \\ \xrightarrow[t^3=5^{6x}]{t=5^{2x}} 625t + 3t^3 = 62500 \Rightarrow 3t^3 + 625t - 62500 = 0 \\ \Rightarrow (t - 25) \cdot (3t^2 + 75t + 2500) = 0 \\ \Rightarrow \begin{cases} t - 25 = 0 \Rightarrow t = 25 = 5^2 = 5^{2x} \Rightarrow 2 = 2x \Rightarrow \boxed{x = 1} \\ 3t^2 + 75t + 2500 = 0 \Rightarrow \nexists \text{ Solución} \end{cases}$$

$$\bullet \quad \begin{array}{c|ccc|c} & 3 & 0 & 625 & -62500 \\ 25 & & 75 & 1875 & 62500 \\ \hline & 3 & 75 & 2500 & 0 \end{array}$$

$$(46) \quad 4^{x-2} - 2^{x+1} = -12 \Rightarrow 2^{2 \cdot (x-2)} - 2 \cdot 2^x = -12 \Rightarrow 2^{2x-4} - 2 \cdot 2^x = -12 \\ \Rightarrow \frac{2^{2x}}{2^4} - 2 \cdot 2^x = -12 \Rightarrow 2^{2x} - 2^5 \cdot 2^x = -12 \cdot 2^4 \xrightarrow[2^{2x}=t^2]{2^x=t} t^2 - 32t = -192 \\ \Rightarrow t^2 - 32t + 192 = 0 \Rightarrow \begin{cases} t = 8 = 2^3 = 2^x \Rightarrow \boxed{x = 3} \\ t = 24 = 2^x \Rightarrow \boxed{x = \log_2 24} \end{cases}$$

$$(47) \quad 3^{2 \cdot (x+2)} - 4 \cdot 3^x - 77 = 0 \Rightarrow 3^{2x+4} - 4 \cdot 3^x - 77 = 0 \Rightarrow 3^4 \cdot 3^{2x} - 4 \cdot 3^x - 77 = 0 \\ \xrightarrow[3^{2x}=t^2]{3^x=t} 81t^2 - 4t - 77 = 0 \Rightarrow \begin{cases} t = -\frac{77}{81} = 3^x \Rightarrow \nexists \text{ Solución} \\ t = 1 = 3^x \Rightarrow \boxed{x = 0} \end{cases}$$

6.1. Ecuaciones exponenciales

$$(48) \quad 3^{2x+5} = 27^{x+2} \implies 3^{2x+5} = 3^{3 \cdot (x+2)} \implies 3^{2x+5} = 3^{3x+6} \implies 2x+5 = 3x+6 \implies \boxed{x = -1}$$

$$(49) \quad 2^{2x} - 5 \cdot 2^x + 4 = 0 \xrightarrow[2^{2x}=t^2]{2^x=t} t^2 - 5t + 4 = 0 \implies \begin{cases} t = 1 = 2^x \implies \boxed{x = 0} \\ t = 4 = 2^2 = 2^x \implies \boxed{x = 2} \end{cases}$$

$$(50) \quad 3^{1+x} + \left(\frac{1}{3}\right)^{-x} = 36 \implies 3 \cdot 3^x + 3^x = 36 \implies 4 \cdot 3^x = 36 \implies 3^x = 9 \implies 3^x = 3^2 \implies \boxed{x = 2}$$

$$(51) \quad 2^{1-x} - \left(\frac{1}{2}\right)^x = 4 \implies \frac{2}{2^x} - \frac{1}{2^x} = 4 \implies 2 - 1 = 4 \cdot 2^x \implies 2^x = \frac{1}{4} \implies 2^x = 2^{-2} \implies \boxed{x = -2}$$

$$(52) \quad 2^x + 2^{1-x} = 3 \implies 2^x + \frac{2}{2^x} = 3 \implies 2^{2x} + 2 = 3 \cdot 2^x \xrightarrow[2^{2x}=t^2]{2^x=t} t^2 + 2 = 3t \implies t^2 - 3t + 2 = 0 \implies \begin{cases} t = 1 = 2^x \implies \boxed{x = 0} \\ t = 2 = 2^x \implies \boxed{x = 1} \end{cases}$$

$$(53) \quad 3^x + \frac{1}{3^{1+x}} = \frac{28}{9} \implies 3^x + \frac{1}{3 \cdot 3^x} = \frac{28}{9} \implies 9 \cdot 3^{2x} + 3 = 28 \cdot 3^x \xrightarrow[3^{2x}=t^2]{3^x=t} 9t^2 + 3 = 28t \implies 9t^2 - 28t + 3 = 0 \implies \begin{cases} t = \frac{1}{9} = 3^{-2} = 3^x \implies \boxed{x = -2} \\ t = 3 = 3^x \implies \boxed{x = 1} \end{cases}$$

$$(54) \quad 4^{7x-1} = 8000 \implies \frac{4^{7x}}{4} = 8000 \implies 4^{7x} = 32000 \implies 4^{7x} = 4^4 \cdot 5^3 \implies \log_4(4^{7x}) = \log_4(4^4 \cdot 5^3) \implies 7x = \log_4(4^4 \cdot 5^3) \implies 7x = \log_4 4^4 + \log_4 5^3 \implies \boxed{x = \frac{4 + \log_4 125}{7}}$$

$$(55) \quad 3^{2x+1} - 12 \cdot 3^x + 3^2 = 0 \implies 3 \cdot 3^{2x} - 12 \cdot 3^x + 3^2 = 0 \xrightarrow[3^{2x}=t^2]{3^x=t} 3t^2 - 12t + 9 = 0 \implies \begin{cases} t = 1 = 3^x \implies \boxed{x = 0} \\ t = 3 = 3^x \implies \boxed{x = 1} \end{cases}$$

$$(56) \quad 3^{x+1} - 3^x - 2 \cdot 3^{x-1} = 12 \implies 3 \cdot 3^x - 3^x - \frac{2 \cdot 3^x}{3} = 12 \implies 9 \cdot 3^x - 3 \cdot 3^x - 2 \cdot 3^x = 36 \implies 4 \cdot 3^x = 36 \implies 3^x = 9 \implies 3^x = 3^2 \implies \boxed{x = 2}$$

$$(57) \quad 5^{x^2-3x} = 625 \implies 5^{x^2-3x} = 5^4 \implies x^2 - 3x = 4 \implies x^2 - 3x - 4 = 0 \implies \begin{cases} \boxed{x = -1} \\ \boxed{x = 4} \end{cases}$$

- 58) $2^{x^2+3} = 200 \Rightarrow 2^3 \cdot 2^{x^2} = 200 \Rightarrow 2^{x^2} = 25 \Rightarrow \log_2 2^{x^2} = \log_2 25$
 $\Rightarrow x^2 = \log_2 25 \Rightarrow \boxed{x = \pm \sqrt{\log_2 25}}$
- 59) $3^{x^2+x+1} = 2187 \Rightarrow 3^{x^2+x+1} = 3^7 \Rightarrow x^2 + x + 1 = 7 \Rightarrow x^2 + x - 6 = 0$
 $\Rightarrow \begin{cases} \boxed{x = -3} \\ \boxed{x = 2} \end{cases}$
- 60) $3^{-x} + 3^{x+1} = 4 \Rightarrow \frac{1}{3^x} + 3 \cdot 3^x = 4 \xrightarrow[3^{2x}=t^2]{3^x=t} \frac{1}{t} + 3t - 4 = 0 \Rightarrow 1 + 3t^2 - 4t = 0$
 $\Rightarrow 3t^2 - 4t + 1 = 0 \Rightarrow \begin{cases} t = \frac{1}{3} = 3^{-1} = 3^x \Rightarrow \boxed{x = -1} \\ t = 1 = 3^x \Rightarrow \boxed{x = 0} \end{cases}$
- 61) $3^{2x+3} = 2187 \Rightarrow 3^{2x+3} = 3^7 \Rightarrow 2x + 3 = 7 \Rightarrow \boxed{x = 2}$
- 62) $\Rightarrow 2^{3x} - 2^{2x} - 4 = 0 \xrightarrow[2^{3x}=t^3]{2^{2x}=t^2} t^3 - t^2 - 4 = 0 \xrightarrow{\bullet} (t-2) \cdot (t^2 + t + 2) = 0$
 $\Rightarrow \begin{cases} t - 2 = 0 \Rightarrow t = 2 = 2^x \Rightarrow \boxed{x = 1} \\ t^2 + t + 2 = 0 \Rightarrow \nexists \text{ Solución} \end{cases} \quad \bullet \quad \begin{array}{ccc|c} 1 & -1 & 0 & -4 \\ 2 & & 2 & 2 \\ \hline 1 & 1 & 2 & 0 \end{array}$
- 63) $3^{x-1} + 3^x - 3^{x+1} = -45 \Rightarrow \frac{3^x}{3} + 3^x - 3 \cdot 3^x = -45 \Rightarrow 3^x + 3 \cdot 3^x - 9 \cdot 3^x = -135$
 $\Rightarrow -5 \cdot 3^x = -135 \Rightarrow 3^x = 27 \Rightarrow 3^x = 3^3 \Rightarrow \boxed{x = 3}$
- 64) $25^{x+2} = 5^{-x-2} \Rightarrow 5^{2 \cdot (x+2)} = 5^{-x-2} \Rightarrow 2x + 4 = -x - 2 \Rightarrow 3x = -6$
 $\Rightarrow \boxed{x = -2}$
- 65) $4^{x-1} = 2^{x+1} \Rightarrow 2^{2 \cdot (x-1)} = 2^{x+1} \Rightarrow 2x - 2 = x + 1 \Rightarrow \boxed{x = 3}$
- 66) $3^{x+2} = 9 \Rightarrow 3^{x+2} = 3^2 \Rightarrow x + 2 = 2 \Rightarrow \boxed{x = 0}$
- 67) $2^{x+1} = 4^x \Rightarrow 2^{x+1} = 2^{2x} \Rightarrow x + 1 = 2x \Rightarrow \boxed{x = 1}$
- 68) $3^{x+1} + 3^{x-2} + 3^x + 3^{x-1} = 120 \Rightarrow 3 \cdot 3^x + \frac{3^x}{3^2} + 3^x + \frac{3^x}{3} = 120$
 $\Rightarrow 27 \cdot 3^x + 3^x + 9 \cdot 3^x + 3 \cdot 3^x = 1080 \Rightarrow 40 \cdot 3^x = 1080 \Rightarrow 3^x = 27$
 $\Rightarrow 3^x = 3^3 \Rightarrow \boxed{x = 3}$
- 69) $\frac{1}{4^x - 2^x} = -1 \Rightarrow 1 = -2^{2x} + 2^x \xrightarrow[2^{2x}=t^2]{2^x=t} 1 = -t^2 + 2t \Rightarrow t^2 - 2t + 1 = 0 \Rightarrow$
 $t = 1 = 2^x \Rightarrow \boxed{x = 0}$
- 70) $8^x - 3 \cdot 4^x - 2^{x+2} + 8 = 0 \Rightarrow 2^{3x} - 3 \cdot 2^{2x} - 2^2 \cdot 2^x = 0 \xrightarrow[2^{2x}=t^2]{2^x=t} t^3 - 3t^2 - 4t = 0$

6.1. Ecuaciones exponenciales

$$\Rightarrow t \cdot (t^2 - 3t - 4) \Rightarrow \begin{cases} t = 0 = 2^x \Rightarrow \nexists \text{ Solución} \\ t^2 - 3t - 4 = 0 \Rightarrow \begin{cases} t = -1 = 2^x \Rightarrow \nexists \text{ Solución} \\ t = 4 = 2^2 = 2^x \Rightarrow \boxed{x = 2} \end{cases} \end{cases}$$

$$(71) \quad 2^{2x} - 3 \cdot 2^x - 4 = 0 \xrightarrow[2^{2x}=t^2]{2^x=t} t^2 - 3t - 4 = 0 \Rightarrow \begin{cases} t = -1 = 2^x \Rightarrow \nexists \text{ Solución} \\ t = 4 = 2^2 = 2^x \Rightarrow \boxed{x = 2} \end{cases}$$

$$(72) \quad 3^x + 3^{x-1} - 3^{x-2} = 11 \Rightarrow 3^x + \frac{3^x}{3} - \frac{3^x}{9} = 11 \xrightarrow{3^x=t} t + \frac{t}{3} - \frac{t}{9} = 11 \\ \Rightarrow 9t + 3t - t = 99 \Rightarrow 11t = 99 \Rightarrow t = 9 = 3^2 = 3^x \Rightarrow \boxed{x = 2}$$

$$(73) \quad 2^x + 2^{-x} = \frac{65}{8} \Rightarrow 2^x + \frac{1}{2^x} = \frac{65}{8} \Rightarrow 2^{2x} + 1 = \frac{65 \cdot 2^x}{8} \xrightarrow[2^{2x}=t^2]{2^x=t} t^2 + 1 = \frac{65t}{8} \\ \Rightarrow 8t^2 - 65t + 8 = 0 \Rightarrow \begin{cases} t = \frac{1}{8} = 2^{-3} = 2^x \Rightarrow \boxed{x = -3} \\ t = 8 = 2^3 = 2^x \Rightarrow \boxed{x = 3} \end{cases}$$

$$(74) \quad 2^{2x+1} = 8^{x-1} \Rightarrow 2^{2x+1} = 2^{3 \cdot (x-1)} \Rightarrow 2x+1 = 3 \cdot (x-1) \\ \Rightarrow 2x+1 = 3x-3 \Rightarrow \boxed{x = 4}$$

$$(75) \quad 3^{x-1} = 3^{x^2-1} \Rightarrow x-1 = x^2-1 \Rightarrow x^2-x = 0 \Rightarrow x \cdot (x-1) = 0 \Rightarrow \begin{cases} \boxed{x = 0} \\ \boxed{x = 1} \end{cases}$$

$$(76) \quad \frac{2^{3x+1}}{2^{x^2}} = \frac{4^{x-1}}{2^x} \Rightarrow \frac{2^{3x+1}}{2^{x^2}} = \frac{2^{2 \cdot (x-1)}}{2^x} \Rightarrow 2^{3x+1-x^2} = 2^{2x-2-x} \\ \Rightarrow 3x+1-x^2 = x-2 \Rightarrow x^2-2x-3 = 0 \Rightarrow \begin{cases} \boxed{x = -1} \\ \boxed{x = 3} \end{cases}$$

$$(77) \quad 3 \cdot 27^{x-2} = 9^x \Rightarrow 3 \cdot (3^3)^{x-2} = (3^2)^x \Rightarrow 3 \cdot 3^{3x-6} = 3^{2x} \Rightarrow 3^{3x-5} = 3^{2x} \\ \Rightarrow 3x-5 = 2x \Rightarrow \boxed{x = 5}$$

$$(78) \quad 5^{x+4} = 125^{x-4} \Rightarrow 5^{x+4} = (5^3)^{x-4} \Rightarrow 5^{x+4} = 5^{3x-12} \Rightarrow x+4 = 3x-12 \\ \Rightarrow \boxed{x = 8}$$

$$(79) \quad \frac{1}{81^{6-x}} = 3^{4-x} \Rightarrow 81^{-(6-x)} = 3^{4-x} \Rightarrow (3^4)^{x-6} = 3^{4-x} \Rightarrow 3^{4x-24} = 3^{4-x} \\ \Rightarrow 4x-24 = 4-x \Rightarrow 5x = 28 \Rightarrow \boxed{x = \frac{28}{5}}$$

$$(80) \quad 32^{2x-3} = 2^{x+3} \Rightarrow (2^5)^{2x-3} = 2^{x+3} \Rightarrow 2^{10x-15} = 2^{x+3} \Rightarrow 10x-15 = x+3 \\ \Rightarrow \boxed{x = 2}$$

$$(81) \quad 125^{x+2} = 5^{2x} \Rightarrow (5^3)^{x+2} = 5^{2x} \Rightarrow 5^{3x+6} = 5^{2x} \Rightarrow 3x+6 = 2x \Rightarrow \boxed{x = -6}$$

$$\textcircled{82} \quad 256^x = 4 \cdot 4^{2x-3} \implies (4^4)^x = 4^{2x-3+1} \implies 4^{4x} = 4^{2x-2} \implies 4x = 2x-2 \\ \implies \boxed{x = -1}$$

$$\textcircled{83} \quad 3^{x^3 - \frac{x^2}{10} - \frac{4x}{5} + \frac{3}{10}} = 1 \implies x^3 - \frac{x^2}{10} - \frac{4x}{5} + \frac{3}{10} = 0 \implies 10x^3 - x^2 - 8x + 3 = 0$$

$$\overset{\bullet}{\implies} (x+1) \cdot (10x^2 - 11x + 3) = 0 \implies \begin{cases} x+1=0 \implies \boxed{x = -1} \\ 10x^2 - 11x + 3 = 0 \implies \begin{cases} \boxed{x = 3/5} \\ \boxed{x = 1/2} \end{cases} \end{cases}$$

$$\textcircled{\bullet} \quad -1 \left| \begin{array}{cccc} 10 & -1 & -8 & 3 \\ & -10 & 11 & -3 \\ \hline 10 & -11 & 3 & 0 \end{array} \right.$$

$$\textcircled{84} \quad \frac{2^{x^3-x^2-5x}}{8} = 1 \implies 2^{x^3-x^2-5x} = 8 \implies 2^{x^3-x^2-5x} = 2^3 \implies x^3 - x^2 - 5x = 3$$

$$\implies x^3 - x^2 - 5x - 3 = 0 \overset{\bullet}{\implies} (x+1) \cdot (x^2 - 2x - 3) = 0 \implies \begin{cases} x+1=0 \implies \boxed{x = -1} \\ x^2 - 2x - 3 = 0 \implies \begin{cases} \boxed{x = -1} \\ \boxed{x = 3} \end{cases} \end{cases}$$

$$\textcircled{\bullet} \quad -1 \left| \begin{array}{cccc} 1 & -1 & -5 & -3 \\ & -1 & 2 & 3 \\ \hline 1 & -2 & -3 & 0 \end{array} \right.$$

$$\textcircled{85} \quad 3^{3x} + 5 \cdot 3^{2x-1} - 11 \cdot 3^{x-1} + 1 = 0 \implies 3^{3x} + \frac{5 \cdot 3^{2x}}{3} - \frac{11 \cdot 3^x}{3} + 1 = 0$$

$$\overset{3^x=t}{\implies} t^3 + \frac{5t^2}{3} - \frac{11t}{3} + 1 = 0 \implies 3t^3 + 5t^2 - 11t + 3 = 0 \overset{\bullet}{\implies} (t-1) \cdot (3t^2 + 8t - 3) = 0$$

$$\implies \begin{cases} t = 1 = 3^x \implies \boxed{x = 0} \\ 3t^2 + 8t - 3 = 0 \implies \begin{cases} t = -3 = 3^x \implies \nexists \text{ Solución} \\ t = \frac{1}{3} = 3^{-1} = 3^x \implies \boxed{x = -1} \end{cases} \end{cases}$$

$$\textcircled{\bullet} \quad 1 \left| \begin{array}{cccc} 3 & 5 & -11 & 3 \\ & 3 & 8 & -3 \\ \hline 3 & 8 & -3 & 0 \end{array} \right.$$

$$\textcircled{86} \quad 4 \cdot \left(\frac{1}{2}\right)^{3x} = 9 \cdot 2^{-2x} - 2^{-x+1} \implies 4 \cdot (2^{-1})^{3x} = 9 \cdot 2^{-2x} - 2 \cdot 2^{-x} \overset{2^{-x}=t}{\implies} 4t^3 = 9t^2 - 2t$$

$$\implies 4t^3 - 9t^2 + 2t = 0 \implies t \cdot (t^2 - 9t + 2) = 0$$

$$\implies \begin{cases} t = 0 = 2^{-x} \implies \nexists \text{ Solución} \\ 4t^2 - 9t + 2 = 0 \implies \begin{cases} t = \frac{1}{4} = 2^{-2} = 2^{-x} \implies -2 = -x \implies \boxed{x = 2} \\ t = 2 = 2^{-x} \implies 1 = -x \implies \boxed{x = -1} \end{cases} \end{cases}$$

6.1. Ecuaciones exponenciales

$$\begin{aligned}
 87 \quad & \left(\frac{3}{2}\right)^{3x+1} + 1 = \frac{3^{2x}}{2} + \left(\frac{3}{2}\right)^{x+1} \implies \frac{3}{2} \cdot \left(\frac{3}{2}\right)^{3x} + 1 = \frac{3^{2x}}{2} + \left(\frac{3}{2}\right) \cdot \left(\frac{3}{2}\right)^x \\
 & \xrightarrow{\left(\frac{3}{2}\right)^x = t} \frac{3}{2}t^3 + 1 = t^2 + \frac{3}{2}t \implies 3t^3 - 2t^2 - 3t + 2 = 0 \implies (t-1) \cdot (3t^2 + t - 2) \\
 & \implies \begin{cases} t-1=0 \implies t=1 = \left(\frac{3}{2}\right)^x \implies \boxed{x=0} \\ 3t^2 + t - 2 = 0 \implies \begin{cases} t=-1 = \left(\frac{3}{2}\right)^x \implies \nexists \text{ Solución} \\ t=\frac{2}{3} = \frac{3^{-1}}{2} = \left(\frac{3}{2}\right)^x \implies \boxed{x=-1} \end{cases} \end{cases} \\
 \odot \quad & 1 \left| \begin{array}{cccc} 3 & -2 & -3 & 2 \\ & 3 & 1 & -2 \\ \hline 3 & 1 & -2 & 0 \end{array} \right.
 \end{aligned}$$

HTTPS://APRENDECONMIGOMELON.COM

6.2. Sistemas de ecuaciones exponenciales

$$\textcircled{1} \begin{cases} 3^{x+y} = 81 \\ 3^{y-x} = 9 \end{cases}$$

$$\textcircled{2} \begin{cases} 3^x + 3^y = 36 \\ 3^{y-x} = 3 \end{cases}$$

$$\textcircled{3} \begin{cases} 2^x + 2^y = 20 \\ 2^{y+x} = 64 \end{cases}$$

$$\textcircled{4} \begin{cases} 2^x + 3^y = 7 \\ 2^{x+1} - 3^{2-y} = 5 \end{cases}$$

$$\textcircled{5} \begin{cases} 2^{2x-y} = 32 \\ 3^{x-2y} = 3 \end{cases}$$

$$\textcircled{6} \begin{cases} 3^x \cdot 9^y = 3^8 \\ 2^{x-1} \cdot 2^{y+1} = 2^6 \end{cases}$$

$$\textcircled{7} \begin{cases} 2 \cdot 3^{x+1} - 3^{y-1} = 15 \\ 5 \cdot 3^{x+2} - 3^{y+1} = 108 \end{cases}$$

$$\textcircled{8} \begin{cases} 3 \cdot 2^x - 2 \cdot 3^y = -42 \\ 5 \cdot 2^{x+1} - 4 \cdot 3^{y-1} = 4 \end{cases}$$

$$\textcircled{9} \begin{cases} 2 \cdot 5^x - 2 \cdot 3^{y+2} = 32 \\ 5^x + 3^{y+1} = 28 \end{cases}$$

$$\textcircled{10} \begin{cases} 3^x - 2^{y+1} = 235 \\ 3^{x-1} - 2^{y-1} = 79 \end{cases}$$

$$\textcircled{11} \begin{cases} 2^x + 5^y = 9 \\ 2^{x+2} + 5^{y+1} = 41 \end{cases}$$

$$\textcircled{12} \begin{cases} x - y = 1 \\ 2^x - 2^y = 2 \end{cases}$$

$$\textcircled{13} \begin{cases} 3 \cdot 2^x + 2 \cdot 3^y = 30 \\ 3^{y+2} - 2 \cdot 2^{x-1} = 77 \end{cases}$$

$$\textcircled{14} \begin{cases} 2^{3x-2y} = \frac{1}{2} \\ 3^{y+x} = 27 \end{cases}$$

$$\textcircled{15} \begin{cases} 3^{x+1} - 2^{y+1} = -3 \\ 2^y - 2 \cdot 3^{x+2} = -4 \end{cases}$$

$$\textcircled{16} \begin{cases} 3^{2x+y} = 3^7 \\ 3^{x-2y} = 3 \end{cases}$$

$$\textcircled{17} \begin{cases} \frac{2^{2x-3}}{2^{3y+2}} = 2^8 \\ 3x - 2y = 17 \end{cases}$$

$$\textcircled{18} \begin{cases} 5^x \cdot 25^y = 5^7 \\ 2^{x-1} \cdot 2^{y+2} = 64 \end{cases}$$

$$\textcircled{19} \begin{cases} 2^x + 5^y = 9 \\ 2^{x-1} + 5^{y+1} = 9 \end{cases}$$

$$\textcircled{20} \begin{cases} 3^x - 2^y = 1 \\ 3^{x-1} = 2^{y-2} + 1 \end{cases}$$

$$\textcircled{21} \begin{cases} 2^{x+2y} = 16 \\ 2^{3x-3y} = 8 \end{cases}$$

$$\textcircled{22} \begin{cases} x - y = 2 \\ 2^x - 2^y = 6 \end{cases}$$

$$\textcircled{23} \begin{cases} x - y = 2 \\ 2^x \cdot 2^y = 8 \end{cases}$$

$$\textcircled{24} \begin{cases} 3^{x-2y} = 3 \\ 3^{2x-3y} = 27 \end{cases}$$

$$\textcircled{25} \begin{cases} 2^x + 2^y = 5 \\ 2^x - 3 \cdot 2^y = -3 \end{cases}$$

$$\textcircled{26} \begin{cases} 5^{x-2y} = 1 \\ 5^{2x-3y} = 5 \end{cases}$$

$$\textcircled{27} \begin{cases} 2^x + 5^y = 9 \\ 2^{x+1} + 5^{y+2} = 133 \end{cases}$$

$$\textcircled{28} \begin{cases} x + Y = 5 \\ 2^x - 2^y = 14 \end{cases}$$

$$\textcircled{29} \begin{cases} 7^{2x+3y} = \frac{1}{7} \\ 7^{-4x-5y} = \frac{1}{7} \end{cases}$$

$$\textcircled{30} \begin{cases} 3 \cdot 2^{x-1} - 2^{y-2} = 4 \\ 4 \cdot 2^{x+1} - 3 \cdot 2^y = 8 \end{cases}$$

6.2. Sistemas de ecuaciones exponenciales

$$\textcircled{31} \begin{cases} x - y = 3 \\ 2^x - 2^y = \frac{7}{4} \end{cases}$$

$$\textcircled{34} \begin{cases} 5^{3x-2y} = 3125 \\ 11^{6x-7y} = 14641 \end{cases}$$

$$\textcircled{32} \begin{cases} \log(x+y) + \log(x-y) = \log 33 \\ e^x \cdot e^y = e^{11} \end{cases}$$

$$\textcircled{35} \begin{cases} 3^{2x+y} = 3^7 \\ 3^{x-2y} = 3 \end{cases}$$

$$\textcircled{33} \begin{cases} 2^x + 5^y = 9 \\ 2^{x-1} + 5^{y+1} = 9 \end{cases}$$

$$\textcircled{36} \begin{cases} 2^x + 5^y = 9 \\ 2^{x-1} + 5^{y+1} = 27 \end{cases}$$

[HTTPS://APRENDECONMIGOMELON.COM](https://aprendeconmigmelon.com)

$$① \begin{cases} 3^{x+y} = 81 \\ 3^{y-x} = 9 \end{cases} \Rightarrow \begin{cases} 3^{x+y} = 3^4 \Rightarrow x+y=4 \\ 3^{y-x} = 3^2 \Rightarrow y-x=2 \end{cases} \Rightarrow \boxed{\begin{matrix} x=1 \\ y=3 \end{matrix}}$$

$$② \begin{cases} 3^x + 3^y = 36 \Rightarrow 3^x + 3^{x+1} = 36 \Rightarrow 3^x + 3 \cdot 3^x = 36 \Rightarrow 4 \cdot 3^x = 36 \\ \Rightarrow 3^x = 9 = 3^2 \Rightarrow \boxed{x=2} \\ 3^{y-x} = 3 \Rightarrow y-x=1 \Rightarrow y=x+1 \xrightarrow{x=2} \boxed{y=3} \end{cases}$$

$$③ \begin{cases} 2^x + 2^y = 20 \Rightarrow 2^x + 2^{6-x} = 20 \Rightarrow 2^x + \frac{2^6}{2^x} = 20 \Rightarrow 2^{2x} + 64 = 20 \cdot 2^x \\ \xrightarrow[2^{2x}=t^2]{2^x=t} t^2 + 64 = 20t \Rightarrow t^2 - 20t + 64 = 0 \\ \Rightarrow \begin{cases} t=4=2^2=2^x \Rightarrow \boxed{x=2} \xrightarrow{y=6-x} \boxed{y=4} \\ t=16=2^4=2^x \Rightarrow \boxed{x=4} \xrightarrow{y=6-x} \boxed{y=2} \end{cases} \\ 2^{y+x} = 64 \Rightarrow 2^{y+x} = 2^6 \Rightarrow y+x=6 \Rightarrow y=6-x \end{cases}$$

$$④ \begin{cases} 2^x + 3^y = 7 \xrightarrow[2^x=u]{3^y=v} u+v=7 \Rightarrow v=7-u \\ 2^{x+1} - 3^{2-y} = 5 \Rightarrow 2 \cdot 2^x - \frac{3^2}{3^y} = 5 \xrightarrow[2^x=u]{3^y=v} 2u - \frac{9}{v} = 5 \Rightarrow 2u - \frac{9}{7-u} = 5 \\ \Rightarrow (7-u) \cdot 2u - 9 = 5 \cdot (7-u) \Rightarrow 14u - 2u^2 - 9 = 35 - 5u \\ \Rightarrow 2u^2 - 19u + 44 = 0 \\ \Rightarrow \begin{cases} u=4=2^2=2^x \Rightarrow \boxed{x=2} \xrightarrow{v=7-u} v=3=3^y \Rightarrow \boxed{y=1} \\ u=\frac{11}{2}=2^x \Rightarrow x=\log_2 \frac{11}{2} \xrightarrow{v=7-u} v=\frac{3}{2}=3^y \Rightarrow \boxed{y=\log_3 \frac{3}{2}} \end{cases} \end{cases}$$

$$⑤ \begin{cases} 2^{2x-y} = 32 \Rightarrow 2^{2x-y} = 2^5 \Rightarrow 2x-y=5 \\ 3^{x-2y} = 3 \Rightarrow x-2y=1 \end{cases} \Rightarrow \boxed{\begin{matrix} x=3 \\ y=1 \end{matrix}}$$

$$⑥ \begin{cases} 3^x \cdot 9^y = 3^8 \Rightarrow 3^x + 3^{2y} = 3^8 \Rightarrow 3^{x+2y} = 3^8 \Rightarrow x+2y=8 \\ 2^{x-1} \cdot 2^{y+1} = 2^6 \Rightarrow 2^{x-1+y+1} = 2^6 \Rightarrow x+y=6 \end{cases} \Rightarrow \boxed{\begin{matrix} x=4 \\ y=2 \end{matrix}}$$

$$⑦ \begin{cases} 2 \cdot 3^{x+1} - 3^{y-1} = 15 \Rightarrow 6 \cdot 3^x - \frac{3^y}{3} = 15 \xrightarrow[3^y=v]{3^x=u} 6u - \frac{v}{3} = 15 \Rightarrow 18u - v = 45 \\ 5 \cdot 3^{x+2} - 3^{y+1} = 108 \Rightarrow 45 \cdot 3^x - 3 \cdot 3^y = 108 \xrightarrow[3^y=v]{3^x=u} 45u - 3v = 108 \Rightarrow 15u - v = 36 \end{cases} \\ \Rightarrow \begin{cases} u=3=3^x \Rightarrow \boxed{x=1} \\ v=9=3^2=3^y \Rightarrow \boxed{y=2} \end{cases}$$

$$⑧ \begin{cases} 3 \cdot 2^x - 2 \cdot 3^y = -42 \xrightarrow[3^y=v]{3^x=u} 3u - 2v = -42 \\ 5 \cdot 2^{x+1} - 4 \cdot 3^{y-1} = 4 \Rightarrow 10 \cdot 2^x - \frac{4 \cdot 3^y}{3} = 4 \Rightarrow 10u - \frac{4v}{3} = 4 \Rightarrow 30u - 4v = 12 \end{cases} \\ \Rightarrow \begin{cases} u=4=2^2=2^x \Rightarrow \boxed{x=2} \\ v=27=3^3=3^y \Rightarrow \boxed{y=3} \end{cases}$$

6.2. Sistemas de ecuaciones exponenciales

$$9 \quad \begin{cases} 2 \cdot 5^x - 2 \cdot 3^{y+2} = 32 \implies 2 \cdot 5^x - 2 \cdot 9 \cdot 3^y = 32 \xrightarrow[3^y=v]{5^x=u} 2u - 18v = 32 \\ 5^x + 3^{y+1} = 28 \implies 5^x + 3 \cdot 3^y = 28 \xrightarrow[3^y=v]{5^x=u} u + 3v = 28 \end{cases}$$

$$\implies \begin{cases} u = 25 = 5^2 = 5^x \implies \boxed{x = 2} \\ v = 1 = 3^0 \implies \boxed{y = 0} \end{cases}$$

$$10 \quad \begin{cases} 3^x - 2^{y+1} = 235 \implies 3^x - 2 \cdot 2^y = 235 \xrightarrow[2^y=v]{3^x=u} u - 2v = 235 \\ 3^{x-1} - 2^{y-1} = 79 \implies \frac{3^x}{3} - \frac{2^y}{2} = 79 \xrightarrow[2^y=v]{3^x=u} \frac{u}{3} - \frac{v}{2} = 79 \implies 2u - 3v = 474 \end{cases}$$

$$\implies \begin{cases} u = 243 = 3^5 = 3^x \implies \boxed{x = 5} \\ v = 4 = 2^2 = 2^y \implies \boxed{y = 2} \end{cases}$$

$$11 \quad \begin{cases} 2^x + 5^y = 9 \xrightarrow[5^y=v]{2^x=u} u + v = 9 \\ 2^{x+2} + 5^{y+1} = 41 \implies 4 \cdot 2^x + 5 \cdot 5^y = 41 \xrightarrow[2^y=v]{3^x=u} 4u + 5v = 41 \end{cases}$$

$$\implies \begin{cases} u = 4 = 2^2 = 2^x \implies \boxed{x = 2} \\ v = 5 = 5^1 \implies \boxed{y = 1} \end{cases}$$

$$12 \quad \begin{cases} x - y = 1 \implies x = y + 1 \xrightarrow{y=1} \boxed{x = 2} \\ 2^x - 2^y = 2 \implies 2^{y+1} - 2^y = 2 \implies 2 \cdot 2^y - 2^y = 2 \implies 2^y = 2 \implies \boxed{y = 1} \end{cases}$$

$$13 \quad \begin{cases} 3 \cdot 2^x + 2 \cdot 3^y = 30 \xrightarrow[3^y=v]{2^x=u} 3u + 2v = 30 \\ 3^{y+2} - 2 \cdot 2^{x-1} = 77 \implies 9 \cdot 3^y - \frac{2 \cdot 2^x}{2} = 77 \xrightarrow[3^y=v]{2^x=u} 9v - u = 77 \end{cases}$$

$$\implies \begin{cases} u = 4 = 2^2 = 2^x \implies \boxed{x = 2} \\ v = 9 = 3^2 = 3^y \implies \boxed{y = 2} \end{cases}$$

$$14 \quad \begin{cases} 2^{3x-2y} = \frac{1}{2} \implies 2^{3x-2y} = 2^{-1} \implies 3x - 2y = -1 \\ 3^{y+x} = 27 \implies 3^{y+x} = 3^3 \implies y + x = 3 \end{cases} \implies \begin{cases} x = 1 \\ y = 2 \end{cases}$$

$$15 \quad \begin{cases} 3^{x+1} - 2^{y+1} = -3 \implies 3 \cdot 3^x - 2 \cdot 2^y = -3 \xrightarrow[2^y=v]{3^x=u} 3u - 2v = -3 \\ 2^y - 2 \cdot 3^{x+2} = -4 \implies 2^y - 2 \cdot 9 \cdot 3^x = -4 \xrightarrow[2^y=v]{3^x=u} v - 18u = -4 \end{cases}$$

$$\implies \begin{cases} u = \frac{1}{3} = 3^{-1} = 3^x \implies \boxed{x = -1} \\ v = 2 = 2^1 \implies \boxed{y = 1} \end{cases}$$

$$16 \quad \begin{cases} 3^{2x+y} = 3^7 \implies 2x + y = 7 \\ 3^{x-2y} = 3 \implies x - 2y = 1 \end{cases} \implies \begin{cases} x = 3 \\ y = 1 \end{cases}$$

$$17 \quad \begin{cases} \frac{2^{2x-3}}{2^{3y+2}} = 2^8 \implies 2^{2x-3-(3y+2)} = 2^8 \implies 2x - 3y - 5 = 8 \\ 3x - 2y = 17 \end{cases} \implies \begin{cases} x = 5 \\ y = -1 \end{cases}$$

$$(18) \begin{cases} 5^x \cdot 25^y = 5^7 \Rightarrow 5^x \cdot 5^{2y} = 5^7 \Rightarrow 5^{x+2y} = 5^7 \Rightarrow x+2y=7 \\ 2^{x-1} \cdot 2^{y+2} = 64 \Rightarrow 2^{x-1+y+2} = 2^6 \Rightarrow x+y+1=6 \end{cases} \Rightarrow \boxed{\begin{matrix} x=3 \\ y=2 \end{matrix}}$$

$$(19) \begin{cases} 2^x + 5^y = 9 \xrightarrow[5^y=v]{2^x=u} u+v=9 \\ 2^{x-1} + 5^{y+1} = 9 \Rightarrow \frac{2^x}{2} + 5 \cdot 5^y = 9 \xrightarrow[5^y=v]{2^x=u} \frac{u}{2} + 5v = 9 \Rightarrow u+10v=18 \end{cases}$$

$$\Rightarrow \begin{cases} u=8=2^3=2^x \Rightarrow \boxed{x=3} \\ v=1=5^0=5^y \Rightarrow \boxed{y=1} \end{cases}$$

$$(20) \begin{cases} 3^x - 2^y = 1 \xrightarrow[2^y=v]{3^x=u} u-v=1 \\ 3^{x-1} = 2^{y-2} + 1 \Rightarrow \frac{3^x}{3} = \frac{2^y}{2^2} + 1 \xrightarrow[2^y=v]{3^x=u} \frac{u}{3} = \frac{v}{4} + 1 \Rightarrow 4u-3v=12 \end{cases}$$

$$\begin{cases} u=9=3^2=3^x \Rightarrow \boxed{x=2} \\ v=8=2^3=2^y \Rightarrow \boxed{y=3} \end{cases}$$

$$(21) \begin{cases} 2^{x+2y} = 16 \Rightarrow 2^{x+2y} = 2^4 \Rightarrow x+2y=4 \\ 2^{3x-3y} = 8 \Rightarrow 2^{3x-3y} = 2^3 \Rightarrow 3x-3y=3 \end{cases} \Rightarrow \begin{matrix} x=2 \\ y=1 \end{matrix}$$

$$(22) \begin{cases} x-y=2 \Rightarrow x=2+y \xrightarrow{y=1} \boxed{x=3} \\ 2^x - 2^y = 6 \Rightarrow 2^{2+y} - 2^y = 6 \Rightarrow 4 \cdot 2^y - 2^y = 6 \Rightarrow 3 \cdot 2^y = 6 \Rightarrow 2^y = 2 \Rightarrow \boxed{y=1} \end{cases}$$

$$(23) \begin{cases} x-y=2 \\ 2^x \cdot 2^y = 8 \Rightarrow 2^{x+y} = 2^3 \Rightarrow x+y=3 \end{cases} \Rightarrow \boxed{\begin{matrix} x=5/2 \\ y=1/2 \end{matrix}}$$

$$(24) \begin{cases} 3^{x-2y} = 3 \Rightarrow x-2y=1 \\ 3^{2x-3y} = 27 \Rightarrow 3^{2x-3y} = 3^3 \Rightarrow 2x-3y=3 \end{cases} \Rightarrow \boxed{\begin{matrix} x=3 \\ y=1 \end{matrix}}$$

$$(25) \begin{cases} 2^x + 2^y = 5 \xrightarrow[2^y=v]{2^x=u} u+v=5 \\ 2^x - 3 \cdot 2^y = -3 \xrightarrow[2^y=v]{2^x=u} u-3v=-3 \end{cases} \Rightarrow \begin{cases} u=3=2^x \Rightarrow \boxed{x=\log_2 3} \\ v=2=2^y \Rightarrow \boxed{y=1} \end{cases}$$

$$(26) \begin{cases} 5^{x-2y} = 1 \Rightarrow x-2y=0 \\ 5^{2x-3y} = 5 \Rightarrow 2x-3y=1 \end{cases} \Rightarrow \boxed{\begin{matrix} x=2 \\ y=1 \end{matrix}}$$

$$(27) \begin{cases} 2^x + 5^y = 9 \xrightarrow[5^y=v]{2^x=u} u+v=9 \\ 2^{x+1} + 5^{y+2} = 133 \Rightarrow 2 \cdot 2^x + 25 \cdot 5^y = 133 \Rightarrow 2u+25v=133 \end{cases}$$

$$\Rightarrow \begin{cases} u=4=2^2=2^x \Rightarrow \boxed{x=2} \\ v=5=5^1=5^y \Rightarrow \boxed{y=1} \end{cases}$$

$$(28) \begin{cases} x+y=5 \Rightarrow y=5-x \\ 2^x - 2^y = 14 \Rightarrow 2^x - 2^{5-x} = 14 \Rightarrow 2^x - \frac{2^5}{2^x} = 14 \Rightarrow 2^{2x} - 32 = 14 \cdot 2^x \end{cases}$$

$$\xrightarrow{2^{2x}=t^2} 2^x = t \quad t^2 - 14t - 32 = 0$$

6.2. Sistemas de ecuaciones exponenciales

$$\Rightarrow \begin{cases} t = -2 = 2^x \Rightarrow \nexists \text{ Solución} \\ t = 16 = 2^4 = 2^x \Rightarrow \boxed{x = 4} \xrightarrow{y=5-x} \boxed{y = 1} \end{cases}$$

$$(29) \begin{cases} 7^{2x+3y} = \frac{1}{7} \Rightarrow 7^{2x+3y} = 7^{-1} \Rightarrow 2x+3y = -1 \\ 7^{-4x-5y} = \frac{1}{7} \Rightarrow 7^{-4x-5y} = 7^{-1} \Rightarrow -4x-5y = -1 \end{cases} \Rightarrow \boxed{\begin{matrix} x = 4 \\ y = -3 \end{matrix}}$$

$$(30) \begin{cases} 3 \cdot 2^{x-1} - 2^{y-2} = 4 \Rightarrow \frac{3 \cdot 2^x}{2} - \frac{2^y}{2^2} = 4 \xrightarrow[2^y=v]{2^x=u} \frac{3u}{2} - \frac{v}{4} = 4 \Rightarrow 6u - v = 16 \\ 4 \cdot 2^{x+1} - 3 \cdot 2^y = 8 \Rightarrow 8 \cdot 2^x - 3 \cdot 2^y = 8 \Rightarrow 8u - 3v = 8 \end{cases}$$

$$\Rightarrow \begin{cases} u = 4 = 2^2 = 2^x \Rightarrow \boxed{x = 2} \\ v = 8 = 2^3 = 2^y \Rightarrow \boxed{y = 3} \end{cases}$$

$$(31) \begin{cases} x - y = 3 \Rightarrow x = y + 3 \xrightarrow{y=-2} \boxed{x = 1} \\ 2^x - 2^y = \frac{7}{4} \Rightarrow 2^{y+3} - 2^y = \frac{7}{4} \Rightarrow 8 \cdot 2^y - 2^y = \frac{7}{4} \Rightarrow 2^y = \frac{1}{4} = 2^{-2} \Rightarrow \boxed{y = -2} \end{cases}$$

$$(32) \begin{cases} \log(x+y) + \log(x-y) = \log 33 \Rightarrow \log[(x+y) \cdot (x-y)] = \log 33 \Rightarrow x^2 - y^2 = 33 \\ x^2 - (11-x)^2 = 33 \Rightarrow \cancel{x^2} - 121 + \cancel{x^2} + 22x = 33 \Rightarrow \boxed{x = 7} \\ e^x \cdot e^y = e^{11} \Rightarrow e^{x+y} = e^{11} \Rightarrow x+y = 11 \Rightarrow y = 11-x \xrightarrow{x=7} \boxed{y = 4} \end{cases}$$

$$(33) \begin{cases} 2^x + 5^y = 9 \xrightarrow[5^y=v]{2^x=u} u + v = 9 \\ 2^{x-1} + 5^{y+1} = 9 \Rightarrow \frac{2^x}{2} + 5 \cdot 5^y = 9 \xrightarrow[5^y=v]{2^x=u} \frac{u}{2} + 5v = 9 \Rightarrow u + 10v = 18 \end{cases}$$

$$\Rightarrow \begin{cases} u = 8 = 2^3 = 2^x \Rightarrow \boxed{x = 3} \\ v = 1 = 5^y \Rightarrow \boxed{y = 0} \end{cases}$$

$$(34) \begin{cases} 5^{3x-2y} = 3125 \Rightarrow 5^{3x-2y} = 5^5 \Rightarrow 3x-2y = 5 \\ 11^{6x-7y} = 14641 \Rightarrow 11^{6x-7y} = 11^4 \Rightarrow 6x-7y = 4 \end{cases} \Rightarrow \boxed{\begin{matrix} x = 3 \\ y = 2 \end{matrix}}$$

$$(35) \begin{cases} 3^{2x+y} = 3^7 \Rightarrow 2x+y = 7 \\ 3^{x-2y} = 3 \Rightarrow x-2y = 1 \end{cases} \Rightarrow \boxed{\begin{matrix} x = 3 \\ y = 1 \end{matrix}}$$

$$(36) \begin{cases} 2^x + 5^y = 9 \xrightarrow[5^y=v]{2^x=u} u + v = 9 \\ 2^{x-1} + 5^{y+1} = 27 \Rightarrow \frac{2^x}{2} + 5 \cdot 5^y = 27 \xrightarrow[5^y=v]{2^x=u} \frac{u}{2} + 5v = 27 \Rightarrow u + 10v = 54 \end{cases}$$

$$\Rightarrow \begin{cases} u = 4 = 2^2 = 2^x \Rightarrow \boxed{x = 2} \\ v = 5 = 5^y \Rightarrow \boxed{y = 1} \end{cases}$$

7.1. Definición de logaritmo

7.1.1. Calcula el valor de los siguientes logaritmos:

① $\log_{1/27} \sqrt[5]{9}$

② $\ln \sqrt[3]{\frac{1}{e^5}}$

③ $\log_{1/8} \sqrt[5]{4}$

④ $\log_{\sqrt{1/32}} \sqrt[7]{64}$

⑤ $\log_{1/25} \sqrt[5]{125}$

⑥ $\ln \sqrt[7]{\frac{1}{e^3}}$

⑦ $\log_{\sqrt{1/8}} \sqrt[7]{4}$

⑧ $\log_{1/81} \sqrt[5]{3}$

⑨ $\log \sqrt{0,00001}$

⑩ $\log_{\sqrt{1/125}} \sqrt[7]{25}$

⑪ $\log_{25} \sqrt[5]{\frac{1}{125}}$

⑫ $\ln \sqrt{\left(\frac{1}{e^2}\right)^{-3}}$

7.1. Definición de logaritmo

$$\begin{aligned} \textcircled{1} \log_{1/27} \sqrt[5]{9} = \log_{1/27} 3^{2/5} = a &\implies \left(\frac{1}{27}\right)^a = 3^{2/5} \implies \left(\frac{1}{3^3}\right)^a = 3^{2/5} \implies (3^{-3})^a = 3^{2/5} \\ &\implies 3^{-3a} = 3^{2/5} \implies -3a = \frac{2}{5} \implies \boxed{a = \log_{1/27} \sqrt[5]{9} = -\frac{2}{15}} \end{aligned}$$

$$\textcircled{2} \ln \sqrt[3]{\frac{1}{e^5}} = \ln e^{-5/3} = -\frac{5}{3} \cdot \ln e^1 = -\frac{5}{3}$$

$$\begin{aligned} \textcircled{3} \log_{1/8} \sqrt[5]{4} = \log_{1/8} 2^{2/5} = a &\implies \left(\frac{1}{8}\right)^a = 2^{2/5} \implies \left(\frac{1}{2^3}\right)^a = 2^{2/5} \implies (2^{-3})^a = 2^{2/5} \\ &\implies 2^{-3a} = 2^{2/5} \implies -3a = \frac{2}{5} \implies \boxed{a = \log_{1/8} \sqrt[5]{4} = -\frac{2}{15}} \end{aligned}$$

$$\begin{aligned} \textcircled{4} \log_{\sqrt{1/32}} \sqrt[7]{64} = \log_{\sqrt{1/32}} 2^{6/7} = a &\implies \left(\sqrt{\frac{1}{32}}\right)^a = 2^{6/7} \implies \left(\sqrt{\frac{1}{2^5}}\right)^a = 2^{6/7} \\ &\implies \left(2^{-5/2}\right)^a = 2^{6/7} \implies 2^{-5a/2} = 2^{6/7} \implies -\frac{5a}{2} = \frac{6}{7} \\ &\implies \boxed{a = \log_{\sqrt{1/32}} \sqrt[7]{64} = -\frac{12}{35}} \end{aligned}$$

$$\begin{aligned} \textcircled{5} \log_{1/25} \sqrt[5]{125} = \log_{1/25} 5^{3/5} = a &\implies \left(\frac{1}{25}\right)^a = \frac{3}{5} \implies \left(\frac{1}{5^2}\right)^a = \frac{3}{5} \implies (5^{-2})^a = \frac{3}{5} \\ &\implies 5^{-2a} = \frac{3}{5} \implies -2a = \frac{3}{5} \implies \boxed{a = \log_{1/25} \sqrt[5]{125} = -\frac{3}{10}} \end{aligned}$$

$$\textcircled{6} \ln \sqrt[7]{\frac{1}{e^3}} = \ln \sqrt[7]{e^{-3}} = \ln e^{-3/7} = -\frac{3}{7} \cdot \ln e^1 = -\frac{3}{7}$$

$$\begin{aligned} \textcircled{7} \log_{\sqrt{1/8}} \sqrt[7]{4} = \log_{\sqrt{1/8}} 2^{2/7} = a &\implies \left(\sqrt{\frac{1}{8}}\right)^a = 2^{2/7} \implies \left(\sqrt{\frac{1}{2^3}}\right)^a = 2^{2/7} \\ &\implies \left(\sqrt{2^{-3}}\right)^a = 2^{2/7} \implies \left(2^{-3/2}\right)^a = 2^{2/7} \implies 2^{-3a/2} = 2^{2/7} \\ &\implies -\frac{3a}{2} = \frac{2}{7} \implies \boxed{a = \log_{\sqrt{1/8}} \sqrt[7]{4} = -\frac{4}{21}} \end{aligned}$$

$$\begin{aligned} \textcircled{8} \log_{1/81} \sqrt[5]{3} = \log_{1/81} 3^{1/5} = a &\implies \left(\frac{1}{81}\right)^a = 3^{1/5} \implies \left(\frac{1}{3^4}\right)^a = 3^{1/5} \implies (3^{-4})^a = 3^{1/5} \\ &\implies 3^{-4a} = 3^{1/5} \implies -4a = \frac{1}{5} \implies \boxed{a = \log_{1/81} \sqrt[5]{3} = -\frac{1}{20}} \end{aligned}$$

$$\textcircled{9} \log \sqrt{0,00001} = \log \sqrt{10^{-5}} = \log 10^{-5/2} = -\frac{5}{2} \cdot \log 10^1 = -\frac{5}{2}$$

$$\begin{aligned} \textcircled{10} \log_{\sqrt{1/125}} \sqrt[7]{25} = \log_{\sqrt{1/125}} 5^{2/7} = a &\implies \left(\sqrt{\frac{1}{125}}\right)^a = 5^{2/7} \implies \left(\frac{1}{\sqrt{125}}\right)^a = 5^{2/7} \\ &\implies \left(\frac{1}{5^{3/2}}\right)^a = 5^{2/7} \implies \left(5^{-3/2}\right)^a = 5^{2/7} \implies 5^{-3a/2} = 5^{2/7} \end{aligned}$$

$$\Rightarrow -\frac{3a}{2} = \frac{2}{7} \Rightarrow \boxed{a = \log_{\sqrt{1/125}} \sqrt[7]{25} = -\frac{4}{21}}$$

$$\textcircled{11} \log_{25} \sqrt[5]{\frac{1}{125}} = \log_{25} \sqrt[5]{\frac{1}{5^3}} = \log_{25} \sqrt[5]{5^{-3}} = \log_{25} 5^{-3/5} = a \Rightarrow 25^a = 5^{-3/5}$$

$$\Rightarrow 5^{2a} = 5^{-3/5} \Rightarrow 2a = -\frac{3}{5} \Rightarrow \boxed{a = \log_{25} \sqrt[5]{\frac{1}{125}} = -\frac{3}{10}}$$

$$\textcircled{12} \ln \sqrt{\left(\frac{1}{e^2}\right)^{-3}} = \ln \sqrt{(e^{-2})^{-3}} = \ln \sqrt{e^6} = \ln e^{6/2} = \ln e^3 = 3 \cdot \overset{1}{\ln e} = 3$$

HTTPS://APRENDECONMIGOMELON.COM

7.1. Definición de logaritmo

7.1.2. Halla el valor de x en los siguientes logaritmos:

① $\log_3 9^x = 2$

⑤ $\log_3 9^{x+3} = 3$

② $\log 2^x = \frac{3}{2}$

⑥ $\log 2^{x/2} = \frac{3}{2}$

③ $\ln 3^x = -1$

⑦ $\ln 3^{x+6} = 3$

④ $\log_2 4^{x+4} = -2$

⑧ $\log_3 27^{3x+4} = -2$

[HTTPS://APRENDECONMIGOMELON.COM](https://aprendeconmigmelon.com)

- ① $\log_3 9^x = 2 \implies 3^2 = 9^x \implies 3^2 = (3^2)^x \implies 3^2 = 3^{2x} \implies 2 = 2x \implies \boxed{x = 1}$
- ② $\log 2^x = \frac{3}{2} \implies x \cdot \log 2 = \frac{3}{2} \implies x = \frac{3}{2 \log 2} = \frac{3}{\log 4} = \frac{\log 10^3}{\log 4} \implies \boxed{x = \log_4 10^3}$
- ③ $\ln 3^x = -1 \implies x \cdot \ln 3 = -1 \implies x = \frac{-1}{\ln 3} = \frac{-\ln e}{\ln 3} \implies \boxed{x = -\log_3 e}$
- ④ $\log_2 4^{x+4} = -2 \implies 2^{-2} = 4^{x+4} \implies 2^{-2} = (2^2)^{x+4} \implies 2^{-2} = 2^{2x+8}$
 $\implies -2 = 2x + 8 \implies \boxed{x = -5}$
- ⑤ $\log_3 9^{x+3} = 3 \implies 3^3 = 9^{x+3} \implies 3^3 = (3^2)^{x+3} \implies 3^3 = 3^{2x+6}$
 $\implies 3 = 2x + 6 \implies \boxed{x = -\frac{3}{2}}$
- ⑥ $\log 2^{x/2} = \frac{3}{2} \implies \frac{x}{2} \cdot \log 2 = \frac{3}{2} \implies x = \frac{3}{4 \cdot \log 2} = \frac{\log 10^3}{\log 2^4} = \frac{\log 10^3}{\log 16}$
 $\implies \boxed{x = \log_{16} 10^3}$
- ⑦ $\ln 3^{x+6} = 3 \implies (x+6) \cdot \ln 3 = 3 \implies x+6 = \frac{3}{\ln 3}$
 $\implies x = \frac{\ln e^3}{\ln 3} - 6 = \log_3 e^3 - \log_3 3^6 = \log_3 \left(\frac{e}{3^2}\right)^3 \implies \boxed{x = 3 \log_3 \frac{e}{9}}$
- ⑧ $\log_3 27^{3x+4} = -2 \implies 3^{-2} = 27^{3x+4} \implies 3^{-2} = (3^3)^{3x+4} \implies 3^{-2} = 3^{9x+12}$
 $\implies -2 = 9x + 12 \implies \boxed{x = -\frac{14}{9}}$

7.2. Ecuaciones logarítmicas

$$\textcircled{1} \log \left(x + \frac{3}{2} \right) - \log \left(\frac{3}{2} - x \right) + \log 2 = \log 7 - \log \left(\frac{7}{2} - x \right)$$

$$\textcircled{2} \log x - \log (3x - 2x^2) + 2 \log (2x - 1) = \log (4x - 3)$$

$$\textcircled{3} \frac{\log (35 - x^3)}{\log (5 - x)} = 3$$

$$\textcircled{4} \log_3 (\sqrt{-x} - 4) + 1 = \frac{1}{2} \cdot \log_3 (-x - 16)$$

$$\textcircled{5} \log_2 x^6 - 8 \log_{2x} x = \log_2 x^x$$

$$\textcircled{6} \log (2x + 4) + \log (3x + 1) - \log 4 = 2 \log (8 - x)$$

$$\textcircled{7} \log (x^2 - 1) - \log (4x - 1) + 2 \log 2 = 0$$

$$\textcircled{8} \log (x - 2) + 1 = \log 2 + \frac{\log (7x^2 + 50)}{2}$$

$$\textcircled{9} 2 \log x - \log 8 = \log \frac{x}{2}$$

$$\textcircled{10} 5 \log x = 3 \log x + 2 \log 3$$

$$\textcircled{11} 3 \log \frac{x}{2} + 2 \log \frac{x}{3} = 3 \log x - \log 8$$

$$\textcircled{12} 3 \log x - 2 \log \frac{x}{3} = 2 \log 3 + 2 \log (2x)$$

$$\textcircled{13} \log \frac{x}{5} + 1 = 2 \log x$$

$$\textcircled{14} 2 \log (4x) + \log \frac{x}{2} = 3$$

$$\textcircled{15} \log_2 \sqrt{x} - \log_2 \sqrt{2} = \frac{1}{2}$$

$$\textcircled{16} \log \sqrt[3]{x} - \log 2 = \log \sqrt[3]{2} - \log \sqrt{2x}$$

$$\textcircled{17} \log (2x + 12) - \log (3x - 2) = \log 2$$

$$\textcircled{18} \log x - \log 2 = 2 \log (x - 3)$$

$$\textcircled{19} \frac{3 \log x}{x} + \log \sqrt{x} = \log x^x$$

$$\textcircled{20} \log_x 4 + \log_{2x} 4 = 3$$

$$\textcircled{21} \log (x - 1) + \log (x - 2) - \log 3 = 2 \log (x - 3)$$

$$\textcircled{22} 2 \log (x - 1) - \log (x + 2) = \log (x + 1) - \log 2$$

- 23 $\log(5x + 4) - \log 2 = \log \sqrt{x + 4}$
- 24 $\log(2x + 14) - \log 2 = \log \sqrt{x + 5} + \log 3$
- 25 $\log \sqrt{3x - 2} - \log \sqrt{2x - 3} = 1 - \log 5$
- 26 $\log \sqrt{3x + 4} + \log \sqrt{5x + 1} = 1 + \log 3$
- 27 $\frac{\log(x^2 - 3)}{\log(x + 3)} = 2$
- 28 $\frac{\log(4 - x^2)}{\log(2 + x)} = 2$
- 29 $\frac{\log 2 + \log(x + 2)}{\log(x + 1)} = 2$
- 30 $\frac{1 + \log_2(x + 6)}{\log_2(x + 2)} = 2$
- 31 $\frac{1 + 2 \log x}{\log x} - \log x = 2$
- 32 $\log_2 \sqrt{x} - \log_2 \sqrt{2} = \frac{1}{\log_2 x}$
- 33 $\log_2 x - \log_4 x = 1$
- 34 $\log_2 x - \log_x 8 = 2$
- 35 $\log_{2x} 16 - \log_x 2 = 0$
- 36 $\log(x - 3) - \log(2x + 1) + 2 \log 3 = 0$
- 37 $\log_2(5x - 2) - 3 \log_2 4 = 5$
- 38 $\frac{\log_2(4x - 3) + \log_2 6}{\log_2(5x + 1)} = 1$
- 39 $2 \log_3(x + 4) - \log_3(5x + 2) = 1$
- 40 $\log_5(7x + 4) - \log_5(x - 2) = 3 - \log_5(x + 2)$
- 41 $2 \log_3(2x - 1) = 3 + \log_3(x - 2)$
- 42 $\log_2(x + 2) + \log_2(5 - 3x) = 2 + \log_2(5 + 3x)$
- 43 $2 \log(3x + 1) - \log(x - 2) = 2$
- 44 $\log 2 + \log(x - 3) = \log \sqrt{2x}$

7.2. Ecuaciones logarítmicas

- 45) $\frac{\log(4+x) + \log(4-x)}{\log(3x+4)} = 2$
- 46) $2\log 2 + \log(x^2 - 1) - \log(4x - 1) = 0$
- 47) $\log(10x - 4) + \log(3x - 4) = 2\log(5x - 2)$
- 48) $\log(x + 1) = 2\log 2 + \log x - \log(3 - x)$
- 49) $\log(x^2 - 1) - \log(3x - 5) - \log(x - 2) - \log(x - 1) = 0$
- 50) $2\log(2x - 2) - \log(x - 1) = 1$
- 51) $2\log x - \log(x - 16) = 2$
- 52) $\log 10^{\sqrt{2x+320}} = 10\sqrt{x}$
- 53) $\log \sqrt{7x + 51} - 1 = \log 9 - \log \sqrt{2x + 67}$
- 54) $\log(x^2 + 21) - \log x = 1$
- 55) $\ln(1 - x) - \ln x = 1$
- 56) $\log(5 - x^2) - \log x = \log x - 1 - \log(x^2 - 2)$
- 57) $\log x - \log(x^2 - 2) = 1 - \log x$
- 58) $\ln x - \ln(1 - x) = 1$
- 59) $\log(7 - x^2) - \log x = 1$
- 60) $\log(2x + 1) - \log x = 2$
- 61) $\log(x + 1) - \log x = 1$
- 62) $\log(x + 3) + \log x = -1$
- 63) $\log(3 - x^2) - \log(2x) = 1$
- 64) $\log x - \log(x - 3) = \log(2x)$
- 65) $\log x - \log(x - 1) = 2$
- 66) $\log(2 + x^2) - \log x = 1$
- 67) $\log x^2 - \log(x + 1) = 1 + \log(x - 1)$
- 68) $\log(3x + 5) - \log x = 2$

$$(69) \log(x+1) + \log(x-1) = \log(25x) - 2$$

$$(70) \log(x+1)^2 - \log x = 1 + \log(2x)$$

$$(71) \log(x+1) + \log(x-1) = 2 + \log x$$

$$(72) \log(x+3) + \log x = 2 \log(x+1)$$

$$(73) \log(4x+1) + \log 2x = 2$$

$$(74) \log(5-x) - \log(x+1) = 2$$

$$(75) \log(3x-1) + \log(x-1) = 1 + \log x$$

$$(76) \log(4x+3) - \log x = 1$$

$$(77) \log(3-x^2) - \log x = 1 + \log(x+2)$$

$$(78) \log(2-x) - \log(x-1) = 2$$

$$(79) \log(5-x^2) - \log x = 1 + \log(x+1)$$

$$(80) \log x - \log(x^2-2) = 1 - \log x$$

$$(81) \log(2x+1) - \log x = 3$$

$$(82) \log x = -1 + \log 5$$

$$(83) \log_5 x = 2 + \log_5 7$$

$$(84) \log_2 x = 3 + \log_2 9$$

$$(85) \log_4 x = 2 + \log_4 \frac{1}{2}$$

$$(86) \log(3x-1) = -2 + \log 50$$

$$(87) \log_2 \frac{3x-1}{4} = 2 + \log_2 \frac{1}{16}$$

$$(88) \log_3 \sqrt{x-5} + \log_3 \sqrt{2x-3} = 1$$

$$(89) \log_2 \sqrt{x} - \log_2 \sqrt[3]{x} = \frac{2}{3}$$

$$(90) \log_{\sqrt{x}} 10 = -\log_x (9-x) = 2$$

7.2. Ecuaciones logarítmicas

$$\textcircled{1} \log\left(x + \frac{3}{2}\right) - \log\left(\frac{3}{2} - x\right) + \log 2 = \log 7 - \log\left(\frac{7}{2} - x\right)$$

$$\Rightarrow \log\left(2 \cdot \frac{\frac{2x+3}{2}}{\frac{3-2x}{2}}\right) = \log\left(\frac{7}{\frac{7-2x}{2}}\right) \Rightarrow \frac{4x+6}{3-2x} = \frac{14}{7-2x}$$

$$\Rightarrow (4x+6) \cdot (7-2x) = 14 \cdot (3-2x) \Rightarrow -8x^2 + 16x + 42 = 42 - 28x$$

$$\Rightarrow 8x^2 - 44x = 0 \Rightarrow \begin{cases} x=0 & \log \frac{3}{2} - \log \frac{3}{2} + \log 2 = \log 7 - \log \frac{7}{2} \checkmark \\ x=\frac{11}{2} & \log 7 - \log(-7) + \dots \end{cases}$$

$$\textcircled{2} \log x - \log(3x - 2x^2) + 2 \log(2x - 1) = \log(4x - 3)$$

$$\Rightarrow \log \frac{x \cdot (2x-1)^2}{3x-2x^2} = \log(4x-3) \Rightarrow \frac{4x^3 - 4x^2 + x}{3x - 2x^2} = 4x - 3$$

$$\Rightarrow 4x^3 - 4x^2 + x = -8x^3 + 18x^2 - 9x \Rightarrow 12x^3 - 22x^2 + 10x = 0$$

$$\Rightarrow 2x \cdot (6x^2 - 11x + 5) = 0 \Rightarrow \begin{cases} x=0 & \log 0 - \dots \\ x=1 & \log 1 - \log 1 + 2 \log 1 = \log 1 \checkmark \\ x=\frac{5}{6} & \log \frac{5}{6} - \log \frac{10}{9} + 2 \log \frac{2}{3} = \log \frac{1}{3} \checkmark \end{cases}$$

$$\textcircled{3} \frac{\log(35 - x^3)}{\log(5 - x)} = 3 \Rightarrow \log(35 - x^3) = 3 \log(5 - x)$$

$$\Rightarrow \log(35 - x^3) = \log(5 - x)^3 \Rightarrow (35 - x^3) = (5 - x)^3$$

$$\Rightarrow 35 - x^3 = -x^3 + 15x^2 - 75x + 125 \Rightarrow 15x^2 - 75x + 90 = 0$$

$$\Rightarrow \begin{cases} x=2 & \frac{\log 27}{\log 3} = 3 \checkmark \\ x=3 & \frac{\log 8}{\log 2} = 3 \checkmark \end{cases}$$

$$\textcircled{4} \log_3(\sqrt{-x} - 4) + 1 = \frac{1}{2} \cdot \log_3(-x - 16) \Rightarrow \log_3(\sqrt{-x} - 4) + \log_3 3 = \log_3 \sqrt{-x - 16}$$

$$\Rightarrow \log_3[3 \cdot (\sqrt{-x} - 4)] = \log_3 \sqrt{-x - 16} \Rightarrow 3 \cdot (\sqrt{-x} - 4) = \sqrt{-x - 16}$$

$$\Rightarrow -9x + 72\sqrt{-x} + 144 = -x - 16 \Rightarrow 72\sqrt{-x} = 8x - 160 \Rightarrow 9\sqrt{-x} = x - 20$$

$$\Rightarrow -81x = x^2 - 40x + 400 \Rightarrow x^2 + 41x + 400 = 0$$

$$\Rightarrow \begin{cases} x=-25 & \log_3 1 + 1 = \frac{1}{2} \cdot \log_3 9 \checkmark \\ x=-16 & \log_3 0 + \dots \end{cases}$$

$$\textcircled{5} \log_2 x^6 - 8 \log_2 x = \log_2 x^x \Rightarrow \log_2 x^6 - \log_2 x^x = \log_2 x^8$$

$$\Rightarrow \log_2 \frac{x^6}{x^x} = \frac{\log_2 x^8}{\log_2 2^x} \Rightarrow \log_2 x^{6-x} = \frac{\log_2 x^8}{x} \Rightarrow x \cdot \log_2 x^{6-x} = \log_2 x^8$$

$$\Rightarrow \log_2 x^{6x-x^2} = \log_2 x^8 \Rightarrow x^{6x-x^2} = x^8$$

$$\Rightarrow \begin{cases} \cancel{x=0} & \cancel{\log_2 0} - \dots \\ \boxed{x=1} & \log_2 1 - 8 \log_2 1 = \log_2 1 \checkmark \\ 6x - x^2 = 0 \Rightarrow x^2 - 6x + 8 = 0 \Rightarrow \begin{cases} \boxed{x=2} & \log_2 2^6 - 8 \log_4 2 = \log_2 4 \checkmark \\ \boxed{x=4} & \log_2 4^6 - 8 \log_{16} 4 = \log_2 4^4 \checkmark \end{cases} \end{cases}$$

⑥ $\log(2x+4) + \log(3x+1) - \log 4 = 2 \log(8-x)$

$$\Rightarrow \log \frac{\log(2x+4) \cdot (3x+1)}{4} = \log(8-x)^2 \Rightarrow \frac{6x^2 + 14x + 4}{4} = x^2 - 16x + 64$$

$$\Rightarrow 6x^2 + 14x + 4 = 4x^2 - 64x + 256 \Rightarrow 2x^2 + 78x - 252 = 0$$

$$\Rightarrow \begin{cases} \cancel{x=42} & \cancel{\log(-80)} - \dots \\ \boxed{x=3} & \log 10 + \log 10 - \log 4 = 2 \log 5 \checkmark \end{cases}$$

⑦ $\log(x^2-1) - \log(4x-1) + 2 \log 2 = 0 \Rightarrow \log(x^2-1) - \log(4x-1) + \log 2^2 = 0$

$$\Rightarrow \log \frac{2^2 \cdot (x^2-1)}{4x-1} = 0 \Rightarrow \frac{4x^2-4}{4x-1} = 1 \Rightarrow 4x^2-4 = 4x-1$$

$$\Rightarrow 4x^2 - 4x - 3 = 0 \Rightarrow \begin{cases} \cancel{x=-\frac{1}{2}} & \cancel{\log\left(-\frac{3}{4}\right)} - \dots \\ \boxed{x=\frac{3}{2}} & \log \frac{5}{4} - \log 5 + 2 \log 2 = 0 \end{cases}$$

⑧ $\log(x-2) + 1 = \log 2 + \frac{\log(7x^2+50)}{2}$

$$\Rightarrow \log(x-2) + \log 10 = \log 2 + \frac{1}{2} \cdot \log(7x^2+50)$$

$$\Rightarrow \log[10 \cdot (x-2)] = \log\left(2 \cdot \sqrt{7x^2+50}\right) \Rightarrow 10x-20 = 2 \cdot \sqrt{7x^2+50}$$

$$\Rightarrow 5x-10 = \sqrt{7x^2+50} \Rightarrow 25x^2-100x+100 = 7x^2+50 \Rightarrow 18x^2-100x+50 = 0$$

$$\Rightarrow \begin{cases} \cancel{x=\frac{5}{9}} & \cancel{\log\left(-\frac{13}{9}\right)} + \dots \\ \boxed{x=5} & \log 3 + 1 = \log 2 + \frac{\log 225}{2} \checkmark \end{cases}$$

⑨ $2 \log x - \log 8 = \log \frac{x}{2} \Rightarrow \log \frac{x^2}{8} = \log \frac{x}{2} \Rightarrow \frac{x^2}{8} = \frac{x}{2} \Rightarrow x^2 = 4x$

$$\Rightarrow x \cdot (x-4) = 0 \Rightarrow \begin{cases} \cancel{x=0} & \cancel{2 \log 0} - \dots \\ \boxed{x=4} & 2 \log 4 - \log 8 = \log 2 \checkmark \end{cases}$$

⑩ $5 \log x = 3 \log x + 2 \log 3 \Rightarrow \log x^5 = \log x^3 3^2 \Rightarrow x^5 = 9x^3 \Rightarrow x^5 - 9x^3 = 0$

$$\Rightarrow x^3 \cdot (x^2-9) = 0 \Rightarrow \begin{cases} \cancel{x=0} & \cancel{5 \log 0} - \dots \\ \cancel{x=-3} & \cancel{5 \log(-3)} - \dots \\ \boxed{x=3} & 5 \log 3 = 3 \log 3 + 2 \log 3 \checkmark \end{cases}$$

7.2. Ecuaciones logarítmicas

$$(11) \quad 3 \log \frac{x}{2} + 2 \log \frac{x}{3} = 3 \log x - \log 8 \implies \log \left[\left(\frac{x}{2} \right)^3 \cdot \left(\frac{x}{3} \right)^2 \right] = \log \frac{x^3}{8}$$

$$\implies \frac{x^5}{72} = \frac{x^3}{8} \implies x^5 = 9x^3 \implies x^5 - 9x^3 = 0 \implies x^3 \cdot (x^2 - 9) = 0$$

$$\implies \begin{cases} x=0 & \log 0 + \dots \\ x=-3 & \log \left(-\frac{3}{2} \right) + \dots \\ \boxed{x=3} & 3 \log \frac{3}{2} + 2 \log 1 = 3 \log 3 - \log 8 \quad \checkmark \end{cases}$$

$$(12) \quad 3 \log x - 2 \log \frac{x}{3} = 2 \log 3 + 2 \log (2x) \implies \log x^3 - \log \left(\frac{x}{3} \right)^2 = \log 3^2 + \log (2x)^2$$

$$\implies \log \frac{x^3}{x^2/9} = \log (36x^2) \implies 9x = 36x^2 \implies 4x^2 - x = 0 \implies x \cdot (4x - 1) = 0$$

$$\implies \begin{cases} x=0 & 3 \log 0 - \dots \\ \boxed{x = \frac{1}{4}} & 3 \log \frac{1}{4} - 2 \log \frac{1}{12} = 2 \log 3 + 2 \log \frac{1}{2} \quad \checkmark \end{cases}$$

$$(13) \quad \log \frac{x}{5} + 1 = 2 \log x \implies \log \frac{x}{5} + \log 10 = 2 \log x \implies \log \frac{10x}{5} = \log x^2 \implies 2x = x^2$$

$$\implies x^2 - 2x = 0 \implies x \cdot (x - 2) = 0 \implies \begin{cases} x=0 & \log 0 + \dots \\ \boxed{x=2} & \log \frac{2}{5} + 1 = 2 \log 2 \quad \checkmark \end{cases}$$

$$(14) \quad 2 \log (4x) + \log \frac{x}{2} = 3 \implies \log \left((4x)^2 \cdot \frac{x}{2} \right) = \log 10^3 \implies 8x^3 = 10^3$$

$$\implies x^3 = 125 \implies \boxed{x=5} \quad 2 \log 20 + \log \frac{5}{2} = 3 \quad \checkmark$$

$$(15) \quad \log_2 \sqrt{x} - \log_2 \sqrt{2} = \frac{1}{2} \implies \log_2 \frac{\sqrt{x}}{\sqrt{2}} = \log_2 \sqrt{2} \implies \frac{\sqrt{x}}{\sqrt{2}} = \sqrt{2} \implies \sqrt{x} = 2$$

$$\implies \boxed{x=4} \quad \log_2 2 - \log_2 \sqrt{2} = \frac{1}{2} \quad \checkmark$$

$$(16) \quad \log \sqrt[3]{x} - \log 2 = \log \sqrt[3]{2} - \log \sqrt{2x} \implies \log \frac{\sqrt[3]{x}}{2} = \log \frac{\sqrt[3]{2}}{\sqrt{2x}} \implies \frac{\sqrt[3]{x}}{2} = \frac{\sqrt[3]{2}}{\sqrt{2x}}$$

$$\implies \frac{x^2}{2^6} = \frac{2^2}{(2x)^3} \implies x^5 = 2^5 \implies \boxed{x=2} \quad \log \sqrt[3]{2} - \log 2 = \log \sqrt[3]{2} - \log 2 \quad \checkmark$$

$$(17) \quad \log (2x + 12) - \log (3x - 2) = \log 2 \implies \log \frac{2x + 12}{3x - 2} = \log 2 \implies \frac{2x + 12}{3x - 2} = 2$$

$$\implies 2x + 12 = 6x - 4 \implies 4x = 16 \implies \boxed{x=4} \quad \log 20 - \log 10 = \log 2 \quad \checkmark$$

$$(18) \quad \log x - \log 2 = 2 \log (x - 3) \implies \log \frac{x}{2} = \log (x - 3)^2 \implies \frac{x}{2} = x^2 - 6x + 9$$

$$\implies 2x^2 - 13x + 18 = 0 \implies \begin{cases} \boxed{x = \frac{9}{2}} & \log \frac{9}{2} - \log 2 = 2 \log \frac{3}{2} \quad \checkmark \\ \boxed{x=2} & \log 2 - \log 2 = 2 \log 1 \quad \checkmark \end{cases}$$

$$(19) \frac{3 \log x}{x} + \log \sqrt{x} = \log x^x \Rightarrow \frac{3}{x} \cdot \log x + \log x^{1/2} = \log x^x \Rightarrow \log x^{\frac{3}{x} + \frac{1}{2}} = \log x^x$$

$$\Rightarrow \frac{3}{x} + \frac{1}{2} = x \Rightarrow 2x^2 - x - 6 = 0 \Rightarrow \begin{cases} x = 0 & \frac{3 \log 0}{0} + \dots \\ \boxed{x = 1} & \frac{3 \log 2}{1} + \log 1 = \log 1 \\ x = -\frac{3}{2} & \frac{3 \log \left(-\frac{3}{2}\right)}{-3/2} + \dots \\ \boxed{x = 2} & \frac{3 \log 2}{2} + \log \sqrt{2} = \log 2^2 \checkmark \end{cases}$$

$$(20) \log_x 4 + \log_{2x} 4 = 3 \Rightarrow \frac{\log_2 4}{\log_2 x} + \frac{\log_2 4}{\log_2 2x} = 3 \xrightarrow{\odot} \frac{2}{z-1} + \frac{2}{z} = 3$$

$$\Rightarrow 2z + 2 \cdot (z-1) = 3z \cdot (z-1) \Rightarrow 2z + 2z - 2 = 3z^2 - 3z \Rightarrow 3z^2 - 7z - 2 = 0$$

$$\Rightarrow \begin{cases} z = 2 \xrightarrow{y=z-1} y = 1 \xrightarrow{x=2^y} \boxed{x = 2} & \log_2 4 + \log_4 4 = 3 \checkmark \\ z = \frac{1}{3} \xrightarrow{y=z-1} y = -\frac{2}{3} \xrightarrow{x=2^y} x = \frac{1}{\sqrt[3]{2}} \Rightarrow \boxed{x = \frac{\sqrt[3]{2}}{2}} & \log_{\sqrt[3]{2}/2} 4 + \log_{\sqrt[3]{2}} 4 = 3 \checkmark \end{cases}$$

$$\odot \left\{ \begin{array}{l} \log_2 x = y \Rightarrow x = 2^y \\ \log_2 2x = z \Rightarrow 2x = 2^z \Rightarrow x = \frac{2^z}{2} \Rightarrow x = 2^{z-1} \end{array} \right\} \Rightarrow 2^y = 2^{z-1} \Rightarrow y = z - 1$$

$$(21) \log(x-1) + \log(x-2) - \log 3 = 2 \log(x-3) \Rightarrow \log \frac{(x-1) \cdot (x-2)}{3} = \log(x-3)^2$$

$$\Rightarrow \frac{x^2 - 3x + 2}{2} = x^2 - 6x + 9 \Rightarrow x^2 - 3x + 2 = 3x^2 - 18x + 27 \Rightarrow 2x^2 - 15x + 25 = 0$$

$$\Rightarrow \begin{cases} x = \frac{5}{2} & \log \frac{3}{2} + \log \frac{1}{2} - \log 3 = 2 \log \left(-\frac{1}{2}\right) \\ \boxed{x = 5} & \log 4 + \log 3 - \log 3 = 2 \log 2 \checkmark \end{cases}$$

$$(22) 2 \log(x-1) - \log(x+2) = \log(x+1) - \log 2 \Rightarrow \log \frac{(x-1)^2}{x+2} = \log \frac{x+1}{2}$$

$$\Rightarrow \frac{x^2 - 2x + 1}{x+2} = \frac{x+1}{2} \Rightarrow 2x^2 - 2x + 1 = (x+1) \cdot (x+2) \Rightarrow 2x^2 - 4x + 2 = x^2 + 3x + 2$$

$$\Rightarrow x^2 - 7x = 0 \Rightarrow x \cdot (x-7) = 0 \Rightarrow \begin{cases} x = 0 & 2 \log 1 - \log 2 = \log 1 - \log 2 \\ \boxed{x = 7} & 2 \log 6 - \log 9 = \log 8 - \log 2 \checkmark \end{cases}$$

$$(23) \log(5x+4) - \log 2 = \log \sqrt{x+4} \Rightarrow \log \frac{5x+4}{2} = \log \sqrt{x+4}$$

$$\Rightarrow \frac{5x+4}{2} = \sqrt{x+4} \Rightarrow 25x^2 + 40x + 16 = 4 \cdot (x+4) \Rightarrow 25x^2 + 36x = 0$$

$$\Rightarrow x \cdot (25x + 36) = 0 \Rightarrow \begin{cases} \boxed{x = 0} & \log 4 - \log 2 = \log \sqrt{4} \checkmark \\ x = -\frac{36}{25} & \log \left(-\frac{16}{5}\right) - \dots \end{cases}$$

7.2. Ecuaciones logarítmicas

- 24) $\log(2x+14) - \log 2 = \log \sqrt{x+5} + \log 3 \Rightarrow \log \frac{2x+14}{2} = \log(3\sqrt{x+5})$
 $\Rightarrow \frac{2x+14}{2} = 3\sqrt{x+5} \Rightarrow 4x^2 + 56x + 196 = 36(x+5) \Rightarrow 4x^2 + 20x + 16 = 0$
 $\Rightarrow \begin{cases} \boxed{x = -1} & \log 12 - \log 2 = \log \sqrt{4} + \log 3 \\ \boxed{x = -4} & \log 6 - \log 2 = \log \sqrt{1} + \log 3 \end{cases}$
- 25) $\log \sqrt{3x-2} - \log \sqrt{2x-3} = 1 - \log 5 \Rightarrow \log \frac{\sqrt{3x-2}}{\sqrt{2x-3}} = \log 10 - \log 5$
 $\Rightarrow \log \frac{\sqrt{3x-2}}{\sqrt{2x-3}} = \log \frac{10}{5} \Rightarrow \frac{\sqrt{3x-2}}{\sqrt{2x-3}} = 2 \Rightarrow 3x-2 = 4(2x-3)$
 $\Rightarrow 5x = 10 \Rightarrow \boxed{x = 2} \quad \log \sqrt{4} - \log \sqrt{1} = 1 - \log 5 \checkmark$
- 26) $\log \sqrt{3x+4} + \log \sqrt{5x+1} = 1 + \log 3 \Rightarrow \log(\sqrt{3x+4} \cdot \sqrt{5x+1}) = \log 10 + \log 3$
 $\Rightarrow \log \sqrt{15x^2 + 23x + 4} = \log 30 \Rightarrow \sqrt{15x^2 + 23x + 4} = 30 \Rightarrow 15x^2 + 23x + 4 = 900$
 $\Rightarrow 15x^2 + 23x - 896 = 0 \Rightarrow \begin{cases} x = \frac{128}{15} & \log \sqrt{\frac{108}{5}} + \dots \\ \boxed{x = 7} & \log \sqrt{25} + \log \sqrt{36} = 1 + \log 3 \checkmark \end{cases} \Rightarrow$
- 27) $\frac{\log(x^2-3)}{\log(x+3)} = 2 \Rightarrow \log(x^2-3) = 2 \log(x+3) \Rightarrow \log(x^2-3) = \log(x+3)^2$
 $\Rightarrow x^2-3 = x^2+6x+9 \Rightarrow 6x = -12 \Rightarrow x = -2 \quad \frac{\log 1}{\log 1} = \dots \Rightarrow \nexists \text{ Solución}$
- 28) $\frac{\log(4-x^2)}{\log(2+x)} = 2 \Rightarrow \log(4-x^2) = 2 \log(2+x) \Rightarrow \log(4-x^2) = \log(2+x)^2$
 $\Rightarrow 4-x^2 = 4+2x+x^2 \Rightarrow 2x^2+2x=0$
 $\Rightarrow 2x \cdot (x+1) = 0 \Rightarrow \begin{cases} \boxed{x = 0} & \frac{\log 4}{\log 2} = 2 \checkmark \\ \boxed{x = -1} & \frac{\log 3}{\log 1} = \dots \end{cases}$
- 29) $\frac{\log 2 + \log(x+2)}{\log(x+1)} = 2 \Rightarrow \log[2 \cdot (x+2)] = 2 \log(x+1)$
 $\Rightarrow \log(2x+4) = \log(x+1)^2 \Rightarrow 2x+4 = x^2+2x+1 \Rightarrow x^2 = 3$
 $\Rightarrow \begin{cases} x = -\sqrt{3} & \frac{\log 2 + \log(2-\sqrt{3})}{\log(1-\sqrt{3})} = \dots \\ \boxed{x = \sqrt{3}} & \frac{\log 2 + \log 2 + \sqrt{3}}{\log 1 + \sqrt{3}} = 2 \checkmark \end{cases}$
- 30) $\frac{1 + \log_2(x+6)}{\log_2(x+2)} = 2 \Rightarrow \log_2 2 + \log_2(x+6) = 2 \log_2(x+2)$
 $\Rightarrow \log_2[2 \cdot (x+6)] = \log_2(x+2)^2 \Rightarrow 2x+12 = x^2+4x+4$

$$\Rightarrow x^2 + 2x - 8 = 0 \Rightarrow \begin{cases} x = -4 & \frac{1 + \log_2 2}{\log_2 0} = \dots \\ x = 2 & \frac{1 + \log_2 8}{\log_2 4} = 2 \checkmark \end{cases}$$

$$(31) \frac{1 + 2 \log x}{\log x} - \log x = 2 \xrightarrow{z = \log x} \frac{1 + 2z}{z} - z = 2 \Rightarrow 1 + 2z - z^2 = 2z$$

$$\Rightarrow z^2 = 1 \Rightarrow \begin{cases} z = 1 = \log x \Rightarrow x = 10 & \frac{1 + 2 \log 10}{\log 10} - \log 10 = 2 \checkmark \\ z = -1 = \log x \Rightarrow x = \frac{1}{10} & \frac{1 + 2 \log \frac{1}{10}}{\log \frac{1}{10}} - \log \frac{1}{10} = 2 \checkmark \end{cases}$$

$$(32) \log_2 \sqrt{x} - \log_2 \sqrt{2} = \frac{1}{\log_2 x} \Rightarrow \frac{1}{2} \cdot \log_2 x - \frac{1}{2} \cdot \log_2 2 = \frac{1}{\log_2 x}$$

$$\xrightarrow{z = \log_2 x} \frac{1}{2}z - \frac{1}{2} = \frac{1}{z} \Rightarrow z^2 - z = 2 \Rightarrow z^2 - z - 2 = 0$$

$$\Rightarrow \begin{cases} z = -1 = \log_2 x \Rightarrow x = \frac{1}{2} & \log_2 \sqrt{\frac{1}{2}} - \log_2 \sqrt{2} = \frac{1}{\log_2 \frac{1}{2}} \checkmark \\ z = 2 = \log_2 x \Rightarrow x = 4 & \log_2 \sqrt{4} - \log_2 \sqrt{2} = \frac{1}{\log_2 4} \checkmark \end{cases}$$

$$(33) \log_2 x - \log_4 x = 1 \Rightarrow \log_2 x - \frac{\log_2 x}{\log_2 4} = 1 \xrightarrow{z = \log_2 x} z - \frac{z}{2} = 1 \Rightarrow 2z - z = 2$$

$$\Rightarrow z = 2 = \log_2 x \Rightarrow x = 4 \quad \log_2 4 - \log_4 4 = 1 \checkmark$$

$$(34) \log_2 x - \log_x 8 = 2 \Rightarrow \log_2 x - \frac{\log_2 8}{\log_2 x} = 2 \xrightarrow{z = \log_2 x} z - \frac{3}{z} = 2 \Rightarrow z^2 - 3 = 2z$$

$$\Rightarrow z^2 - 2z - 3 = 0 \Rightarrow \begin{cases} z = -1 = \log_2 x \Rightarrow x = \frac{1}{2} & \log_2 \frac{1}{2} - \log_{1/2} 8 = 2 \checkmark \\ z = 3 = \log_2 x \Rightarrow x = 8 & \log_2 8 - \log_8 8 = 2 \checkmark \end{cases}$$

$$(35) \log_{2x} 16 - \log_x 2 = 0 \Rightarrow \frac{\log_2 16}{\log_2 2x} - \frac{\log_2 2}{\log_2 x} = 0 \Rightarrow \frac{4}{\log_2 2x} = \frac{1}{\log_2 x}$$

$$\Rightarrow 4 \log_2 x = \log_2 2x \Rightarrow \log_2 x^4 = \log_2 2x \Rightarrow x^4 = 2x \Rightarrow x^4 - 2x = 0 \Rightarrow x(x^3 - 2) = 0$$

$$\Rightarrow \begin{cases} x = 0 & \log_0 16 = \dots \\ x^3 - 2 = 0 \Rightarrow x = \sqrt[3]{2} & \log_{\sqrt[3]{2}} 16 - \log_{\sqrt[3]{2}} 2 = 0 \checkmark \end{cases}$$

$$(36) \log(x - 3) - \log(2x + 1) + 2 \log 3 = 0 \Rightarrow \log(x - 3) - \log(2x + 1) + \log 3^2 = 0$$

$$\Rightarrow \log \frac{(x - 3) \cdot 9}{2x + 1} = 0 \Rightarrow \frac{9x - 27}{2x + 1} = 1 \Rightarrow 9x - 27 = 2x + 1$$

$$\Rightarrow 7x = 28 \Rightarrow x = 4 \quad \log 1 - \log 9 + 2 \log 3 = 0 \checkmark$$

$$(37) \log_2(5x - 2) - 3 \log_2 4 = 5 \Rightarrow \log_2(5x - 2) - \log_2 4^3 = 5 \Rightarrow \log_2 \frac{5x - 2}{4^3} = 5$$

7.2. Ecuaciones logarítmicas

$$\Rightarrow \frac{5x-2}{2^6} = 2^5 \Rightarrow 5x-2 = 2^{11} \Rightarrow \boxed{x=410} \quad \log_2 2048 - 3\log_2 4 = 5 \checkmark$$

$$(38) \quad \frac{\log_2(4x-3) + \log_2 6}{\log_2(5x+1)} = 1 \Rightarrow \log_2[(4x-3) \cdot 6] = \log_2(5x+1)$$

$$\Rightarrow 24x - 18 = 5x + 1 \Rightarrow 19x = 19 \Rightarrow \boxed{x=1} \quad \frac{\log_2 1 + \log_2 6}{\log_2 6} = 1 \checkmark$$

$$(39) \quad 2\log_3(x+4) - \log_3(5x+2) = 1 \Rightarrow \log_3 \frac{(x+4)^2}{5x+2} = 1 \Rightarrow \frac{x^2+8x+16}{5x+2} = 3$$

$$\Rightarrow x^2+8x+16 = 15x+6 \Rightarrow x^2-7x+10 \Rightarrow \begin{cases} \boxed{x=2} & 2\log_3 6 - \log_3 12 = 1 \checkmark \\ \boxed{x=5} & 2\log_3 9 - \log_3 27 \neq 1 \checkmark \end{cases}$$

$$(40) \quad \log_5(7x+4) - \log_5(x-2) = 3 - \log_5(x+2) \Rightarrow \log_5 \frac{7x+4}{x-2} = \log_5 5^3 - \log_5(x+2)$$

$$\Rightarrow \log_5 \frac{7x+4}{x-2} = \log_5 \frac{125}{x+2} \Rightarrow \frac{7x+4}{x-2} = \frac{125}{x+2}$$

$$\Rightarrow (7x+4) \cdot (x+2) = 125 \cdot (x-2) \Rightarrow 7x^2 + 18x + 8 = 125x - 250$$

$$\Rightarrow 7x^2 - 107x + 258 = 0 \Rightarrow \begin{cases} \boxed{x=3} & \log_5 25 - \log_5 1 = 3 - \log_5 5 \checkmark \\ \boxed{x=\frac{86}{7}} & \log_5 90 - \log_5 \frac{72}{7} = 3 - \log_5 \frac{100}{7} \checkmark \end{cases}$$

$$(41) \quad 2\log_3(2x-1) = 3 + \log_3(x-2) \Rightarrow \log_3(2x-1)^2 = \log_3 3^3 + \log_3(x-2)$$

$$\Rightarrow \log_3(2x-1)^2 = \log_3 27 \cdot (x-2) \Rightarrow 4x^2 - 4x + 1 = 27 \cdot (x-2)$$

$$\Rightarrow 4x^2 - 31x + 55 = 0 \Rightarrow \begin{cases} \boxed{x=\frac{11}{4}} & 2\log_3 \frac{9}{2} = 3 + \log_3 \frac{3}{4} \checkmark \\ \boxed{x=5} & 2\log_3 9 = 3 + \log_3 3 \checkmark \end{cases}$$

$$(42) \quad \log_2(x+2) + \log_2(5-3x) = 2 + \log_2(5+3x)$$

$$\Rightarrow \log_2[(x+2) \cdot (5-3x)] = \log_2 2^2 + \log_2(5+3x)$$

$$\Rightarrow \log_2(-3x^2 - x + 10) = \log_2[4 \cdot (5+3x)] \Rightarrow -3x^2 - x + 10 = 20 + 12x$$

$$\Rightarrow 3x^2 + 13x + 10 = 0 \Rightarrow \begin{cases} \cancel{x = -\frac{10}{3}} & \cancel{\log_2 -\frac{4}{3} + \dots} \\ \boxed{x=-1} & \log_2 1 + \log_2 8 = 2 + \log_2 2 \checkmark \end{cases}$$

$$(43) \quad 2\log(3x+1) - \log(x-2) = 2 \Rightarrow \log \frac{(3x+1)^2}{x-2} = 2 \Rightarrow \frac{(3x+1)^2}{x-2} = 10^2$$

$$\Rightarrow 9x^2 + 6x + 1 = 100x - 200 \Rightarrow 9x^2 - 94x + 201 = 0$$

$$\Rightarrow \begin{cases} \boxed{x=3} & 2\log 10 - \log 1 = 2 \checkmark \\ \boxed{x=\frac{67}{9}} & 2\log \frac{70}{3} - \log \frac{49}{9} = 2 \checkmark \end{cases}$$

$$(44) \quad \log 2 + \log(x-3) = \log \sqrt{2x} \Rightarrow \log[2 \cdot (x-3)] = \log \sqrt{2x} \Rightarrow 2x - 6 = \sqrt{2x}$$

$$\Rightarrow 4x^2 - 24x + 36 = 2x \Rightarrow 4x^2 - 26x + 36 = 0 \Rightarrow \begin{cases} \cancel{x=2} & \log 2 + \log(\cancel{-1}) \stackrel{\#}{=} \dots \\ \boxed{x = \frac{9}{2}} & \log 2 + \log \frac{3}{2} = \log \sqrt{9} \checkmark \end{cases}$$

$$\begin{aligned} 45) \quad & \frac{\log(4+x) + \log(4-x)}{\log(3x+4)} = 2 \Rightarrow \log[(4+x) \cdot (4-x)] = 2 \log(3x+4) \\ & \Rightarrow \log 16 - x^2 = \log(3x+4)^2 \Rightarrow 16 - x^2 = 9x^2 + 24x + 16 \Rightarrow 10x^2 + 24x = 0 \\ & \Rightarrow 2x \cdot (5x + 12) = 0 \Rightarrow \begin{cases} \boxed{x=0} & \frac{\log 4 + \log 4}{\log 4} = 2 \checkmark \\ \boxed{x = -\frac{12}{5}} & \frac{\log \frac{8}{5} + \log \frac{32}{5}}{\log\left(-\frac{16}{5}\right)} \neq 2 \checkmark \end{cases} \end{aligned}$$

$$\begin{aligned} 46) \quad & 2 \log 2 + \log(x^2 - 1) - \log(4x - 1) = 0 \Rightarrow \log \frac{[2^2] \cdot (x^2 - 1)}{\log(4x - 1)} = 0 \Rightarrow \frac{4x^2 - 4}{4x - 1} = 1 \\ & \Rightarrow 4x^2 - 4x - 3 = 0 \Rightarrow \begin{cases} \cancel{x = -\frac{1}{2}} & 2 \log 2 + \log\left(\cancel{-\frac{3}{4}}\right) \stackrel{\#}{=} \dots \\ \boxed{x = \frac{3}{2}} & 2 \log 2 + \log \frac{5}{4} - \log 5 = 0 \checkmark \end{cases} \end{aligned}$$

$$\begin{aligned} 47) \quad & \log(10x - 4) + \log(3x - 4) = 2 \log(5x - 2) \Rightarrow \log[(10x - 4) \cdot (3x - 4)] = \log(5x - 2)^2 \\ & \Rightarrow (10x - 4) \cdot (3x - 4) = (5x - 2)^2 \Rightarrow 30x^2 - 52x + 16 = 25x^2 - 20x + 4 \\ & \Rightarrow 5x^2 - 32x + 12 = 0 \Rightarrow \begin{cases} \cancel{x = \frac{2}{5}} & \log 1 + \log\left(\cancel{-\frac{14}{5}}\right) \stackrel{\#}{=} \dots \\ \boxed{x = 6} & \log 56 + \log 14 = 2 \log 28 \checkmark \end{cases} \end{aligned}$$

$$\begin{aligned} 48) \quad & \log(x+1) = 2 \log 2 + \log x - \log(3-x) \Rightarrow \log(x+1) = \log 2^2 + \log x - \log(3-x) \\ & \Rightarrow \log(x+1) = \log \frac{4x}{3-x} \Rightarrow x+1 = \frac{4x}{3-x} \Rightarrow (x+1) \cdot (3-x) = 4x \\ & \Rightarrow -x^2 + 2x + 3 = 4x \Rightarrow x^2 + 2x - 3 = 0 \Rightarrow \begin{cases} \cancel{x = -3} & \log 4 = 2 \log 2 + \log \cancel{3} \stackrel{\#}{=} \log 6 \\ \boxed{x = 1} & \log 2 = 2 \log 2 + \log 1 - \log 2 \checkmark \end{cases} \end{aligned}$$

$$\begin{aligned} 49) \quad & \log(x^2 - 1) - \log(3x - 5) - \log(x - 2) - \log(x - 1) = 0 \\ & \Rightarrow \log \frac{x^2 - 1}{(3x - 5) \cdot (x - 2) \cdot (x - 1)} = 0 \Rightarrow \log \frac{(\cancel{x-1}) \cdot (x+1)}{(3x - 5) \cdot (x - 2) \cdot (\cancel{x-1})} = 0 \\ & \Rightarrow \frac{(x+1)}{(3x - 5) \cdot (x - 2)} = 1 \Rightarrow x+1 = (3x-5) \cdot (x-2) \Rightarrow x+1 = 3x^2 - 11x + 10 \\ & \Rightarrow 3x^2 - 12x + 9 = 0 \Rightarrow \begin{cases} \cancel{x=1} & \log \cancel{0} \stackrel{\#}{=} \log \cancel{2} \stackrel{\#}{=} \log \cancel{1} \stackrel{\#}{=} \log \cancel{0} = 0 \\ \boxed{x=3} & \log 8 - \log 4 - \log 1 - \log 2 = 0 \checkmark \end{cases} \end{aligned}$$

$$50) \quad 2 \log(2x - 2) - \log(x - 1) = 1 \Rightarrow 2 \log[2 \cdot (x - 1)] - \log(x - 1) = 1$$

7.2. Ecuaciones logarítmicas

$$\Rightarrow \log 4 \cdot (x-1)^2 - \log (x-1) = 1 \Rightarrow \log \frac{4 \cdot (x-1)^2}{x-1} = 1$$

$$\Rightarrow 4 \cdot (x-1) = 10 \Rightarrow 4x - 4 = 10 \Rightarrow \boxed{x = \frac{7}{2}} \quad 2 \log 5 - \log \frac{7}{2} = 1 \checkmark$$

$$(51) \quad 2 \log x - \log (x-16) = 2 \Rightarrow \log x^2 - \log (x-16) = 2 \Rightarrow \log \frac{x^2}{x-16} = 2$$

$$\Rightarrow \frac{x^2}{x-16} = 10^2 \Rightarrow x^2 - 100x + 1600 = 0 \Rightarrow \begin{cases} \boxed{x = 20} & 2 \log 20 - \log 4 = 2 \checkmark \\ \boxed{x = 80} & 2 \log 80 - \log 64 = 2 \checkmark \end{cases}$$

$$(52) \quad \log 10^{\sqrt{2x+320}} = 10\sqrt{x} \Rightarrow \sqrt{2x+320} \cdot \log 10^1 = 10\sqrt{x} \Rightarrow 2x+320 = 100x$$

$$\Rightarrow 98x = 320 \Rightarrow \boxed{x = \frac{160}{49}} \quad \log 10^{\frac{40\sqrt{10}}{7}} = \frac{40\sqrt{10}}{7} \checkmark$$

$$(53) \quad \log \sqrt{7x+51} - 1 = \log 9 - \log \sqrt{2x+67}$$

$$\Rightarrow \log \sqrt{7x+51} - \log 10 = \log 9 - \log \sqrt{2x+67}$$

$$\Rightarrow \log \frac{\sqrt{7x+51}}{10} = \log \frac{9}{\sqrt{2x+67}} \Rightarrow \frac{\sqrt{7x+51}}{10} = \frac{9}{\sqrt{2x+67}}$$

$$\Rightarrow \frac{7x+51}{100} = \frac{81}{2x+67} \Rightarrow 14x^2 + 571x - 4683 = 0$$

$$\Rightarrow \begin{cases} \boxed{x = 7} & \log \sqrt{100} - 1 = \log 9 - \log \sqrt{81} \checkmark \\ x = -669/14 & \log \sqrt{\frac{-567}{7}} - 1 = \log 9 - \log \sqrt{\frac{-200}{7}} \end{cases}$$

$$(54) \quad \log (x^2 + 21) - \log x = 1 \Rightarrow \log \frac{x^2 + 21}{x} = 1 \Rightarrow \frac{x^2 + 21}{x} = 10$$

$$\Rightarrow x^2 + 21 = 10x \Rightarrow x^2 - 10x + 21 = 0 \Rightarrow \begin{cases} \boxed{x = 3} & \log 30 - \log 3 = 1 \checkmark \\ \boxed{x = 7} & \log 70 - \log 7 = 1 \checkmark \end{cases}$$

$$(55) \quad \ln (1-x) - \ln x = 1 \Rightarrow \ln \frac{1-x}{x} = 1 \Rightarrow \frac{1-x}{x} = e \Rightarrow 1-x = ex$$

$$\Rightarrow 1 = ex + x \Rightarrow 1 = x \cdot (e+1) \Rightarrow \boxed{x = \frac{1}{e+1}} \quad \ln \frac{e}{e+1} - \ln \frac{1}{e+1} = 1 \checkmark$$

$$(56) \quad \log (5-x^2) - \log x = \log x - 1 - \log (x^2 - 2) \Rightarrow \log \frac{5-x^2}{x} = \log \frac{x-1}{x^2-2}$$

$$\Rightarrow \frac{5-x^2}{x} = \frac{x-1}{x^2-2} \Rightarrow (5-x^2) \cdot (x^2-2) = x^2-x \Rightarrow -x^4 + 7x^2 - 10 = x^2 - x$$

$$\Rightarrow x^4 - 6x^2 - x + 10 = 0 \stackrel{\odot}{\Rightarrow} (x-2) \cdot (x^3 + 2x^2 - 2x - 5) = 0$$

$$\Rightarrow \boxed{x = 2} \quad \log 1 - \log 2 = \log 1 - \log 2 \checkmark$$

$$\odot \quad \begin{array}{c|cccc|c} & 1 & 0 & -6 & -1 & 10 \\ 2 & & 2 & 4 & -4 & -10 \\ \hline & 1 & 2 & -2 & -5 & 0 \end{array}$$

$$(57) \log x - \log(x^2 - 2) = 1 - \log x \implies 2 \log x - \log(x^2 - 2) = 1$$

$$\implies \log \frac{x^2}{x^2 - 2} = 1 \implies \frac{x^2}{x^2 - 2} = 10 \implies x^2 = 10x^2 - 20 \implies 9x^2 = 20$$

$$\implies \begin{cases} x = \frac{2\sqrt{5}}{3} & \log\left(\frac{2\sqrt{5}}{3}\right) - \dots \\ \boxed{x = \frac{2\sqrt{5}}{3}} & 2 \log \frac{2\sqrt{5}}{3} - \log \frac{2}{9} = 1 \checkmark \end{cases}$$

$$(58) \ln x - \ln(1 - x) = 1 \implies \ln \frac{x}{1 - x} = 1 \implies \frac{x}{1 - x} = e \implies x = e - ex$$

$$\implies x + ex = e \implies x \cdot (1 + e) = e \implies \boxed{x = \frac{e}{1 + e}} \quad \ln \frac{e}{1 + e} - \ln \frac{1}{1 + e} = 1 \checkmark$$

$$(59) \log(7 - x^2) - \log x = 1 \implies \log \frac{7 - x^2}{x} = 1 \implies \frac{7 - x^2}{x} = 10 \implies 7 - x^2 = 10x$$

$$\implies x^2 + 10x - 7 = 0 \implies \begin{cases} x = -5 - 4\sqrt{2} & \log(-50 - 40\sqrt{2}) - \log(-5 - 4\sqrt{2}) = 1 \\ \boxed{x = -5 + 4\sqrt{2}} & \log(-50 + 40\sqrt{2}) - \log(-5 + 4\sqrt{2}) = 1 \checkmark \end{cases}$$

$$(60) \log(2x + 3) + \log x = -1 \implies \log[(2x + 3) \cdot x] = -1 \implies (2x + 3) \cdot x = 10^{-1}$$

$$\implies 2x^2 + 3x = \frac{1}{10} \implies 20x^2 + 30x - 1 = 0$$

$$\implies \begin{cases} x = \frac{-15 - 7\sqrt{5}}{20} & \log \frac{15 - 7\sqrt{5}}{10} + \log \frac{-15 - 7\sqrt{5}}{20} = -1 \\ \boxed{x = \frac{-15 + 7\sqrt{5}}{20}} & \log \frac{15 + 7\sqrt{5}}{10} + \log \frac{-15 + 7\sqrt{5}}{20} = -1 \checkmark \end{cases}$$

$$(61) \log(x + 1) - \log x = 1 \implies \log \frac{x + 1}{x} = 1 \implies \frac{x + 1}{x} = 10 \implies x + 1 = 10x$$

$$\implies \boxed{x = \frac{1}{9}} \quad \log \frac{10}{9} - \log \frac{1}{9} = 1 \checkmark$$

$$(62) \log(2x + 1) - \log x = 2 \implies \log \frac{2x + 1}{x} = 2 \implies \frac{2x + 1}{x} = 10^2 \implies 2x + 1 = 100x$$

$$\implies \boxed{x = \frac{1}{98}} \quad \log \frac{100}{98} - \log \frac{1}{98} = 2 \checkmark$$

$$(63) \log(3 - x^2) - \log(2x) = 1 \implies \log \frac{3 - x^2}{2x} = 1 \implies \frac{3 - x^2}{2x} = 10 \implies x^2 + 20x - 3 = 0$$

$$\implies \begin{cases} x = -10 - \sqrt{103} & \log(-200 - 20\sqrt{103}) - \log(-20 - \sqrt{103}) \\ \boxed{x = -10 + \sqrt{103}} & \log(-200 + 20\sqrt{103}) - \log(-20 + 2\sqrt{103}) = 1 \checkmark \end{cases}$$

$$(64) \log x - \log(x - 3) = \log(2x) \implies \log \frac{x}{x - 3} = \log(2x) \implies \frac{x}{x - 3} = 2x$$

7.2. Ecuaciones logarítmicas

$$\Rightarrow x = 2x^2 - 6x \Rightarrow 2x^2 - 7x = 0 \Rightarrow x \cdot (2x - 7) = 0$$

$$\Rightarrow \begin{cases} x = 0 & \log 0 + \dots \\ x = \frac{7}{2} & \log \frac{7}{2} - \log \frac{1}{2} = \log 7 \checkmark \end{cases}$$

$$(65) \log x - \log(x-1) = 2 \Rightarrow \log \frac{x}{x-1} = 2 \Rightarrow \frac{x}{x-1} = 10^2$$

$$\Rightarrow x = 100x - 100 \Rightarrow \boxed{x = \frac{100}{99}} \quad \log \frac{100}{99} - \log \frac{1}{99} = 2 \checkmark$$

$$(66) \log(2+x^2) - \log x = 1 \Rightarrow \log \frac{2+x^2}{x} = 1 \Rightarrow \frac{2+x^2}{x} = 10 \Rightarrow 2+x^2 = 10x$$

$$\Rightarrow x^2 - 10x + 2 = 0 \Rightarrow \begin{cases} x = 5 - \sqrt{23} & \log 50 - 10\sqrt{23} - \log(5 - \sqrt{23}) \checkmark \\ x = 5 + \sqrt{23} & \log 50 + 10\sqrt{23} - \log(5 + \sqrt{23}) \checkmark \end{cases}$$

$$(67) \log x^2 - \log(x+1) = 1 + \log(x-1) \Rightarrow \log \frac{x^2}{x+1} = \log 10 + \log(x-1) \Rightarrow$$

$$\log \frac{x^2}{x+1} = \log 10 \cdot (x-1) \Rightarrow \frac{x^2}{x+1} = 10 \cdot (x-1) \Rightarrow x^2 = 10 \cdot (x-1) \cdot (x+1)$$

$$\Rightarrow x^2 = 10x^2 - 10 \Rightarrow 9x^2 = 10$$

$$\Rightarrow \begin{cases} x = \frac{\sqrt{10}}{3} & \log \frac{10}{9} - \log \frac{3 - \sqrt{10}}{3} \dots \\ x = \frac{\sqrt{10}}{3} & \log \frac{10}{9} - \log \frac{3 + \sqrt{10}}{3} = 1 + \log \frac{\sqrt{10} - 3}{3} \checkmark \end{cases}$$

$$(68) \log(3x+5) - \log x = 2 \Rightarrow \log \frac{3x+5}{x} = 10 \Rightarrow \frac{3x+5}{x} = 10^2$$

$$\Rightarrow 3x+5 = 100x \Rightarrow 97x = 5 \Rightarrow \boxed{x = \frac{5}{97}} \quad \log \frac{500}{97} - \log \frac{5}{97} = 2 \checkmark$$

$$(69) \log(x+1) + \log(x-1) = \log(25x) - 2$$

$$\Rightarrow \log[(x+1) \cdot (x-1)] = \log(25x) - \log 10^2 \Rightarrow \log(x^2 - 1) = \log \frac{25x}{100}$$

$$\Rightarrow x^2 - 1 = \frac{25x}{100} \Rightarrow 100x^2 - 100 = 25x \Rightarrow 4x^2 - x - 4 = 0$$

$$\Rightarrow \begin{cases} x = \frac{1 - \sqrt{65}}{8} & \log \frac{9 - \sqrt{65}}{8} + \log \frac{-7 - \sqrt{65}}{8} = \dots \\ x = \frac{1 + \sqrt{65}}{8} & \log \frac{9 + \sqrt{65}}{8} + \log \frac{-7 + \sqrt{65}}{8} = \log \frac{25 + 25\sqrt{65}}{8} - 2 \checkmark \end{cases}$$

$$(70) \log(x+1)^2 - \log x = 1 + \log(2x) \Rightarrow \log \frac{(x+1)^2}{x} = \log 10 + \log(2x)$$

$$\Rightarrow \log \frac{(x+1)^2}{x} = \log 10 \cdot (2x) \Rightarrow \frac{(x+1)^2}{x} = 20x \Rightarrow x^2 + 2x + 1 = 20x^2$$

$$\Rightarrow 19x^2 - 2x - 1 = 0 \Rightarrow \left\{ x = \frac{1 - 2\sqrt{5}}{19} \quad x = \frac{1 + 2\sqrt{5}}{19} \right.$$

$$(71) \log(x+1) + \log(x-1) = 2 + \log x \Rightarrow \log(x^2 - 1) = \log 10^2 + \log x$$

$$\Rightarrow \log x^2 - 1 = \log 100x \Rightarrow x^2 - 1 = 100x \Rightarrow x^2 - 100x - 1 = 0$$

$$\left\{ \begin{array}{l} x = \frac{100 - \sqrt{10000 - 4}}{2} \quad \log(51 - \sqrt{2501}) + \log(49 - \sqrt{2501}) = \dots \\ \boxed{x = 50 + \sqrt{2501}} \quad \log(51 + \sqrt{2501}) + \log(49 + \sqrt{2501}) = 2 + \log(50 + \sqrt{2501}) \checkmark \end{array} \right.$$

$$(72) \log(x+3) + \log x = 2 \log(x+1) \Rightarrow \log(x+3) \cdot x = \log(x+1)^2$$

$$\Rightarrow (x+3) \cdot x = (x+1)^2 \Rightarrow x^2 + 3x = x^2 + 2x + 1 \Rightarrow \boxed{x = 1} \quad \log 4 + \log 1 = 2 \log 2 \checkmark$$

$$(73) \log(4x+1) + \log 2x = 2 \Rightarrow \log(4x+1) \cdot 2x = \log 10^2 \Rightarrow (4x+1) \cdot 2x = 100$$

$$\Rightarrow 8x^2 + 2x - 100 = 0 \Rightarrow \left\{ \begin{array}{l} x = \frac{-1 - \sqrt{801}}{8} \quad \log \frac{1 - \sqrt{801}}{2} + \log \frac{-1 - \sqrt{801}}{4} = 2 \\ \boxed{x = \frac{-1 + \sqrt{801}}{8}} \quad \log \frac{1 + \sqrt{801}}{2} + \log \frac{-1 + \sqrt{801}}{4} = 2 \checkmark \end{array} \right.$$

$$(74) \log(5-x) - \log(x+1) = 2 \Rightarrow \log \frac{5-x}{x+1} = \log 10^2 \Rightarrow \frac{5-x}{x+1} = 100 \Rightarrow 5-x = 100x+100$$

$$\Rightarrow 101x = -95 \Rightarrow x = -\frac{95}{101} \quad \log \frac{600}{101} - \log \frac{6}{95} = 2 \checkmark$$

$$(75) \log(3x-1) + \log(x-1) = 1 + \log x \Rightarrow \log[(3x-1) \cdot (x-1)] = \log 10 + \log x$$

$$\Rightarrow \log(3x^2 - 4x + 1) = \log 10x \Rightarrow 3x^2 - 4x + 1 = 10x \Rightarrow 3x^2 - 14x + 1 = 0$$

$$\Rightarrow \left\{ \begin{array}{l} x = \frac{7 - \sqrt{46}}{3} \quad \log \frac{18 - 3\sqrt{46}}{3} + \dots \\ \boxed{x = \frac{7 + \sqrt{46}}{3}} \quad \log \frac{18 + 3\sqrt{46}}{3} + \log \frac{4 + \sqrt{46}}{3} = 1 + \log \frac{7 + \sqrt{46}}{3} \checkmark \end{array} \right.$$

$$(76) \log(4x+3) - \log x = 1 \Rightarrow \log \frac{4x+3}{x} = \log 10 \Rightarrow \frac{4x+3}{x} = 10$$

$$\Rightarrow 4x+3 = 10x \Rightarrow \boxed{x = \frac{1}{2}} \quad \log 5 - \log \frac{1}{2} = 1 \checkmark$$

$$(77) \log(3-x^2) - \log x = 1 + \log(x+2) \Rightarrow \log \frac{3-x^2}{x} = \log 10 + \log(x+2)$$

$$\Rightarrow \log \frac{3-x^2}{x} = \log [10 \cdot (x+2)] \Rightarrow \frac{3-x^2}{x} = 10 \cdot (x+2)$$

$$\Rightarrow 3 - x^2 = 10x^2 + 20x \Rightarrow 11x^2 + 20x - 3 = 0$$

$$\left\{ \begin{array}{l} x = \frac{-10 - \sqrt{133}}{11} \quad \log \frac{130 - 20\sqrt{133}}{121} - \log \frac{-10 - \sqrt{133}}{11} + \dots \\ \boxed{x = \frac{-10 + \sqrt{133}}{11}} \quad \log \frac{130 + 20\sqrt{133}}{121} - \log \frac{-10 + \sqrt{133}}{11} = 1 + \log \frac{12 + \sqrt{133}}{11} \checkmark \end{array} \right.$$

7.2. Ecuaciones logarítmicas

- 78) $\log(2-x) - \log(x-1) = 2 \Rightarrow \log \frac{2-x}{x-1} = \log 10^2 \Rightarrow \frac{2-x}{x-1} = 100$
 $\Rightarrow 2-x = 100x-100 \Rightarrow 101x = 102 \Rightarrow \boxed{x = \frac{102}{101}} \quad \log \frac{100}{101} - \log \frac{1}{101} = 2 \checkmark$
- 79) $\log(5-x^2) - \log x = 1 + \log(x+1) \Rightarrow \log \frac{5-x^2}{x} = \log 10 + \log(x+1)$
 $\Rightarrow \log \frac{5-x^2}{x} = \log(10x+10) \Rightarrow \frac{5-x^2}{x} = 10x+10 \Rightarrow 5-x^2 = 10x^2+10x$
 $\Rightarrow 11x^2+10x-5=0$
 $\Rightarrow \begin{cases} x = \frac{-5-4\sqrt{5}}{11} & \log \frac{500-40\sqrt{5}}{121} - \log \frac{-5-4\sqrt{5}}{11} = \dots \\ \boxed{x = \frac{-5+4\sqrt{5}}{11}} & \log \frac{500+40\sqrt{5}}{121} - \log \frac{-5+4\sqrt{5}}{11} = 1 + \log \frac{6+4\sqrt{5}}{11} \checkmark \end{cases}$
- 80) $\log x - \log(x^2-2) = 1 - \log x \Rightarrow \log \frac{x}{x^2-2} = \log 10 - \log x$
 $\Rightarrow \log \frac{x}{x^2-2} = \log \frac{10}{x} \Rightarrow \frac{x}{x^2-2} = \frac{10}{x} \Rightarrow x^2 = 10x^2 - 20 \Rightarrow 9x^2 = 20$
 $\Rightarrow \begin{cases} x = \frac{2\sqrt{5}}{3} & \log \left(\frac{2\sqrt{5}}{3} \right) - \dots \\ \boxed{x = \frac{2\sqrt{5}}{3}} & \log \frac{2\sqrt{5}}{3} - \log \frac{2}{9} = 1 - \log \frac{2\sqrt{5}}{3} \checkmark \end{cases}$
- 81) $\log(2x+1) - \log x = 3 \Rightarrow \log \frac{2x+1}{x} = \log 10^3 \Rightarrow \frac{2x+1}{x} = 1000$
 $\Rightarrow 998x = 1 \Rightarrow \boxed{x = \frac{1}{998}} \quad \log \frac{500}{499} - \log \frac{1}{998} = 3 \checkmark$
- 82) $\log x = -1 + \log 5 \Rightarrow \log x - \log 5 = -1 \Rightarrow \log \frac{x}{5} = -1 \Rightarrow \frac{x}{5} = 10^{-1}$
 $\Rightarrow x = 5 \cdot 10^{-1} = \frac{5}{10} \Rightarrow \boxed{x = \frac{1}{2}} \quad \log \frac{1}{2} = -1 + \log 5 \Rightarrow \log \frac{1}{10} = -1 \checkmark$
- 83) $\log_5 x = 2 + \log_5 7 \Rightarrow \log_5 x - \log_5 7 = 2 \Rightarrow \log_5 \frac{x}{7} = 2 \Rightarrow \frac{x}{7} = 5^2$
 $\Rightarrow \boxed{x = 175} \quad \log_5 175 = 2 + \log_5 7 \Rightarrow \log_5 25 = 2 \checkmark$
- 84) $\log_2 x = 3 + \log_2 9 \Rightarrow \log_2 x - \log_2 9 = 3 \Rightarrow \log_2 \frac{x}{9} = 3 \Rightarrow \frac{x}{9} = 2^3$
 $\Rightarrow \boxed{x = 72} \quad \log_2 72 = 3 + \log_2 9 \Rightarrow \log_2 8 = 3 \checkmark$
- 85) $\log_4 x = 2 + \log_4 \frac{1}{2} \Rightarrow \log_4 x - \log_4 \frac{1}{2} = 2 \Rightarrow \log_4 2x = 2 \Rightarrow 2x = 4^2$
 $\Rightarrow \boxed{x = 8} \quad \log_4 8 = 2 + \log_4 \frac{1}{2} \Rightarrow \log_4 16 = 2 \checkmark$
- 86) $\log(3x-1) = -2 + \log 50 \Rightarrow \log 3x - 1 - \log 50 = -2 \Rightarrow \log \frac{3x-1}{50} = -2$

$$\Rightarrow \frac{3x-1}{50} = 10^{-2} \Rightarrow 3x-1 = \frac{50}{100} \Rightarrow 3x = \frac{3}{2} \Rightarrow \boxed{x = \frac{1}{2}}$$

$$\log\left(\frac{3}{2} - 1\right) = -2 + \log 50 \Rightarrow \log \frac{1}{2} = -2 + \log 50 \Rightarrow \log \frac{1}{100} = -2 \checkmark$$

$$(87) \log_2 \frac{3x-1}{4} = 2 + \log_2 \frac{1}{16} \Rightarrow \log_2 \frac{3x-1}{4} - \log_2 \frac{1}{16} = 2 \Rightarrow \log_2 \frac{16^4 \cdot (3x-1)}{4} = 2$$

$$\Rightarrow 12x - 4 = 2^2 \Rightarrow \boxed{x = \frac{2}{3}} \quad \log_2 \frac{1}{4} = 2 + \log_2 2^{-4} \checkmark$$

$$(88) \log_3 \sqrt{x-5} + \log_3 \sqrt{2x-3} = 1 \Rightarrow \log_3 (\sqrt{x-5} \cdot \sqrt{2x-3}) = 1$$

$$\Rightarrow \sqrt{(x-5) \cdot (2x-3)} = 3 \Rightarrow \sqrt{2x^2 - 13x + 15} = 3 \Rightarrow 2x^2 - 13x + 15 = 9$$

$$\Rightarrow 2x^2 - 13x + 6 = 0 \Rightarrow \begin{cases} \boxed{x = 6} & \log_3 1 + \log_3 3 = 1 \checkmark \\ x = \frac{1}{2} & \log_3 \sqrt{-\frac{9}{2}} + \dots \end{cases}$$

$$(89) \log_2 \sqrt{x} - \log_2 \sqrt[3]{x} = \frac{2}{3} \Rightarrow \log_2 \frac{\sqrt{x}}{\sqrt[3]{x}} = \frac{2}{3} \Rightarrow \sqrt[6]{\frac{x^3}{x^2}} = 2^{2/3} \Rightarrow x = (2^{2/3})^6 = 2^{12/3}$$

$$\Rightarrow \boxed{x = 16} \quad \log_2 4 - \log_2 \sqrt[3]{16} = \frac{2}{3} \Rightarrow 2 - \frac{1}{3} \log_2 16 = \frac{2}{3} \checkmark$$

$$(90) \log_{\sqrt{x}} 10 - \log_x (9-x) = 2 \xRightarrow{\bullet} \log_x 100 - \log_x (9-x) = 2 \Rightarrow \log_x \frac{100}{9-x} = 2$$

$$\Rightarrow \frac{100}{9-x} = x^2 \Rightarrow 100 = 9x^2 - x^3 \Rightarrow x^3 - 9x^2 + 100 = 0$$

$$\xRightarrow{(*)} \begin{cases} \boxed{x = 5} & \log_{\sqrt{5}} 10 - \log_5 4 = 2 \Rightarrow 2 \log_5 10 - \log_5 4 = 2 \checkmark \\ x^2 - 4x - 20 = 0 \Rightarrow \begin{cases} x = 2 - \sqrt{6} & \log_{\sqrt{2-\sqrt{6}}} 10 - \dots \\ x = 2 + \sqrt{6} & \log_{\sqrt{2+\sqrt{6}}} 10 - \log_{2+\sqrt{6}} (7 - \sqrt{6}) \neq 2 \end{cases} \end{cases}$$

$$\bullet \log_{\sqrt{x}} 10 = \frac{\log_x 10}{\log_x \sqrt{x}} = \frac{\log_x 10}{\log_x x^{1/2}} = \frac{\log_x 10}{\frac{1}{2} \log_x x} = 2 \log_x 10 = \log_x 10^2 = \log_x 100$$

$$(*) \quad 5 \left| \begin{array}{cccc} 1 & -9 & 0 & 100 \\ & 5 & -20 & -100 \\ \hline 1 & -4 & -20 & 0 \end{array} \right.$$

7.3. Sistemas de ecuaciones logarítmicas

$$\textcircled{1} \begin{cases} x + y = 110 \\ \log x + \log y = 3 \end{cases}$$

$$\textcircled{2} \begin{cases} \log_3 x - \log_3 y = 1 \\ x - 2y = 9 \end{cases}$$

$$\textcircled{3} \begin{cases} -3x + y = 70 \\ \log y - \log x^2 = 0 \end{cases}$$

$$\textcircled{4} \begin{cases} x + y = 12 \\ \log_2 x - \log_2 y^3 = -3 \end{cases}$$

$$\textcircled{5} \begin{cases} \log_2 (x - y) = 2 \\ \log_2 x - \log_2 y = 1 \end{cases}$$

$$\textcircled{6} \begin{cases} \log x + \log y = 4 \\ y - 4x = 0 \end{cases}$$

$$\textcircled{7} \begin{cases} \log x + \log y = 3 \\ x - 3y = 70 \end{cases}$$

$$\textcircled{8} \begin{cases} \log x + \log y = 3 \\ \log x - \log y = 1 \end{cases}$$

$$\textcircled{9} \begin{cases} x^2 - y^2 = 11 \\ \log x - \log y = 1 \end{cases}$$

$$\textcircled{10} \begin{cases} \frac{\log x}{\log y} = \frac{1}{2} \\ \log x^2 + \log y = 4 \end{cases}$$

$$\textcircled{11} \begin{cases} \log_x (y - 72) = 2 \\ \log_y (x + 6) = \frac{1}{2} \end{cases}$$

$$\textcircled{12} \begin{cases} \log_2 (x - y) = 2 \\ \log_2 x - \log_2 y = 1 \end{cases}$$

$$\textcircled{13} \begin{cases} \log_2 x^3 - \log_2 y = 3 \\ \log_2 2x - \log_2 y^2 = 2 \end{cases}$$

$$\textcircled{14} \begin{cases} \log x + \log y = 3 \\ 2 \log x - 2 \log y = -2 \end{cases}$$

$$\textcircled{15} \begin{cases} x - y = 21 \\ \log x + \log y = 2 \end{cases}$$

$$\textcircled{16} \begin{cases} \log x + 3 \log y = 5 \\ \log \frac{x}{y} = 1 \end{cases}$$

$$\textcircled{17} \begin{cases} 3x + 2y = 64 \\ \log x - \log y = 1 \end{cases}$$

$$\textcircled{18} \begin{cases} x + y = 30 \\ \log_3 x - \log_3 y = 3 \end{cases}$$

$$\textcircled{19} \begin{cases} \log x + 3 \log y = 5 \\ \log x - \log y = 3 \end{cases}$$

$$\textcircled{20} \begin{cases} \log x + 5 \log y = 7 \\ \log \frac{x}{y} = 1 \end{cases}$$

$$\textcircled{21} \begin{cases} 2 \log x + \log y = 5 \\ \log xy = 4 \end{cases}$$

$$\textcircled{22} \begin{cases} \log x + \log y^3 = 5 \\ \log \frac{x^2}{y} = 3 \end{cases}$$

$$\textcircled{23} \begin{cases} y - 4x = 0 \\ \log x + \log y = 4 \end{cases}$$

$$\textcircled{24} \begin{cases} 2 \log x - 3 \log y = 5 \\ 3 \log x + \log y = 2 \end{cases}$$

$$\textcircled{25} \begin{cases} x - 2y = 9 \\ \log_3 x - \log_3 y = 1 \end{cases}$$

$$\textcircled{26} \begin{cases} x - y = 3 \\ \log x + \log y = 1 \end{cases}$$

$$\textcircled{27} \begin{cases} 5^{x+y} = 5^{11} \\ \log (x + y) + \log (x - y) = \log 33 \end{cases}$$

$$\begin{aligned} ① \quad & \begin{cases} x + y = 110 \implies y = 110 - x \\ \log x + \log y = 3 \implies \log xy = 3 \implies xy = 1000 \xrightarrow{y=110-x} x \cdot (110 - x) = 1000 \end{cases} \\ & \implies x^2 - 110x + 1000 = 0 \implies \begin{cases} \boxed{x = 10} \xrightarrow{y=110-x} \boxed{y = 100} \checkmark \\ \boxed{x = 100} \xrightarrow{y=110-x} \boxed{y = 10} \checkmark \end{cases} \end{aligned}$$

$$② \quad \begin{cases} \log_3 x - \log_3 y = 1 \implies \log \frac{x}{y} = 1 \implies \frac{x}{y} = 3 \implies x = 3y \\ x - 2y = 9 \implies x = 9 + 2y \xrightarrow{x=3y} 3y = 9 + 2y \implies \boxed{y = 9} \xrightarrow{x=3y} \boxed{x = 27} \checkmark \end{cases}$$

$$\begin{aligned} ③ \quad & \begin{cases} -3x + y = 70 \implies y = 70 + 3x \\ \log y - \log x^2 = 0 \implies \log \frac{y}{x^2} = 0 \implies \frac{y}{x^2} = 1 \implies y = x^2 \xrightarrow{y=70+3x} x^2 = 70 + 3x \end{cases} \\ & \implies x^2 - 3x - 70 = 0 \\ & \implies \begin{cases} \boxed{x = -7} \xrightarrow{y=70+3x} \boxed{y = 49} & \log 49 - \log (-7)^2 = 0 \checkmark \\ \boxed{x = 10} \xrightarrow{y=70+3x} \boxed{y = 100} & \log 100 - \log 10^2 = 0 \checkmark \end{cases} \end{aligned}$$

$$\begin{aligned} ④ \quad & \begin{cases} x + y = 12 \implies x = 12 - y \\ \log_2 x - \log_2 y^3 = -3 \implies \log_2 \frac{x}{y^3} = -3 \implies \frac{x}{y^3} = 2^{-3} \implies x = \frac{y^3}{8} \end{cases} \\ & \xrightarrow{x=12-y} 12 - y = \frac{y^3}{8} \implies y^3 + 8y - 96 = 0 \\ & \xRightarrow{\bullet} (y - 4) \cdot (y^2 + 4y + 24) \\ & \implies \begin{cases} \boxed{y = 4} \xrightarrow{x=12-y} \boxed{x = 8} & \log_2 8 - \log_2 4^3 = -3 \checkmark \\ y^2 + 4y + 24 = 0 \implies \nexists \text{ Solución} \end{cases} \\ & \bullet \quad \begin{array}{ccc|c} 1 & 0 & 8 & -96 \\ 4 & 4 & 16 & 96 \\ 1 & 4 & 24 & 0 \end{array} \end{aligned}$$

$$⑤ \quad \begin{cases} \log_2 (x - y) = 2 \\ \log_2 x - \log_2 y = 1 \end{cases}$$

$$\begin{aligned} ⑥ \quad & \begin{cases} \log x + \log y = 4 \implies \log (xy) = 4 \implies xy = 10^4 \xrightarrow{y=4x} x \cdot 4x = 10000 \\ \implies \begin{cases} \cancel{x = 50} \xrightarrow{y=4x} \cancel{y = 200} & \log(\cancel{50}) + \dots \\ \boxed{x = 50} \xrightarrow{y=4x} \boxed{y = 200} & \log 50 + \log 200 = 4 \checkmark \end{cases} \\ y - 4x = 0 \implies y = 4x \end{cases} \end{aligned}$$

$$\begin{aligned} ⑦ \quad & \begin{cases} \log x + \log y = 3 \implies \log (xy) = 3 \implies xy = 1000 \xrightarrow{x=70+3y} (70 + 3y) \cdot y = 1000 \\ \implies 3y^2 + 70y - 1000 = 0 \\ \implies \begin{cases} \cancel{y = \frac{100}{3}} \xrightarrow{x=70+3y} \cancel{x = 30} & \log(\cancel{30}) + \dots \\ \boxed{y = 10} \xrightarrow{x=70+3y} \boxed{x = 100} & \log 100 + \log 10 = 3 \checkmark \end{cases} \\ x - 3y = 70 \implies x = 70 + 3y \end{cases} \end{aligned}$$

7.3. Sistemas de ecuaciones logarítmicas

$$\textcircled{8} \begin{cases} \log x + \log y = 3 \\ \log x - \log y = 1 \end{cases} \Rightarrow 2 \log x = 4 \Rightarrow \log x = 2 \Rightarrow \boxed{x = 100}$$

$$\Rightarrow \log 100 + \log y = 3 \Rightarrow \log y = 1 \Rightarrow \boxed{y = 10}$$

$$\begin{cases} \log 100 + \log 10 = 3 \checkmark \\ \log 100 - \log 10 = 1 \checkmark \end{cases}$$

$$\textcircled{9} \begin{cases} x^2 - y^2 = 11 \xrightarrow{x=10y} (10y)^2 - y^2 = 11 \Rightarrow 99y^2 = 11 \Rightarrow y^2 = \frac{1}{9} \\ \Rightarrow \begin{cases} y = -\frac{1}{3} \xrightarrow{x=10y} x = -\frac{10}{3} \quad \log\left(-\frac{10}{3}\right) - \log\left(-\frac{1}{3}\right) = \dots \\ \boxed{y = \frac{1}{3}} \xrightarrow{x=10y} \boxed{x = \frac{10}{3}} \quad \log \frac{10}{3} - \log \frac{1}{3} = 1 \checkmark \end{cases} \\ \log x - \log y = 1 \Rightarrow \log \frac{x}{y} = 1 \Rightarrow \frac{x}{y} = 10 \Rightarrow x = 10y \end{cases}$$

$$\textcircled{10} \begin{cases} \frac{\log x}{\log y} = \frac{1}{2} \Rightarrow \log y = 2 \log x \Rightarrow \log y = \log x^2 \\ \log x^2 + \log y = 4 \xrightarrow{\log y = \log x^2} \log y + \log y = 4 \Rightarrow 2 \log y = 4 \Rightarrow \log y = 2 \\ \Rightarrow \boxed{y = 100} \xrightarrow{\log y = \log x^2} 2 = \log x^2 \Rightarrow 2 = 2 \log x \Rightarrow \log x = 1 \\ \Rightarrow \boxed{x = 10} \begin{cases} \frac{\log 10}{\log 100} = \frac{1}{2} \checkmark \\ \log 10^2 + \log 100 = 4 \checkmark \end{cases} \end{cases}$$

$$\textcircled{11} \begin{cases} \log_x (y - 72) = 2 \Rightarrow x^2 = y - 72 \xrightarrow{(x+6)^2=y} x^2 = (x+6)^2 - 72 \Rightarrow x^2 = x^2 + 12x - 36 \\ \Rightarrow \boxed{x = 3} \xrightarrow{(x+6)^2=y} \boxed{y = 81} \\ \begin{cases} \log_3 (81 - 72) = 2 \checkmark \\ \log_{81} (3 + 6) = \frac{1}{2} \checkmark \end{cases} \\ \log_y (x + 6) = \frac{1}{2} \Rightarrow 2 \log_y (x + 6) = 1 \Rightarrow \log_y (x + 6)^2 = 1 \Rightarrow (x + 6)^2 = y \end{cases}$$

$$\textcircled{12} \begin{cases} \log_2 (x - y) = 2 \Rightarrow x - y = 2^2 \Rightarrow x = 4 + y \\ \log_2 x - \log_2 y = 1 \Rightarrow \log_2 \frac{x}{y} = 1 \Rightarrow \frac{x}{y} = 2 \Rightarrow x = 2y \Rightarrow \boxed{\begin{matrix} x = 8 \\ y = 4 \end{matrix}} \\ \begin{cases} \log_2 (8 - 4) = 2 \checkmark \\ \log_2 8 - \log_2 4 = 1 \checkmark \end{cases} \end{cases}$$

$$\textcircled{13} \begin{cases} \log_2 x^3 - \log_2 y = 3 \Rightarrow \log_2 \frac{x^3}{y} = 3 \Rightarrow \frac{x^3}{y} = 2^3 \Rightarrow x^3 = 8y \\ \log_2 2x - \log_2 y^2 = 2 \Rightarrow \log_2 \frac{2x}{y^2} = 2 \Rightarrow \frac{2x}{y^2} = 2^2 \Rightarrow x = 2y^2 \\ \Rightarrow (2y^2)^3 = 8y \Rightarrow 8y^6 = 8y \Rightarrow \boxed{y = 1} \xrightarrow{x=2y^2} \boxed{y = 2} \end{cases}$$

$$\begin{cases} \log_2 2^3 - \log_2 1 = 3 \checkmark \\ \log_2 4 - \log_2 1^2 = 2 \checkmark \end{cases}$$

$$\begin{aligned} 14 \quad & \begin{cases} \log x + \log y = 3 \Rightarrow \log(xy) = 3 \Rightarrow xy = 10^3 \xrightarrow{y=10x} x \cdot 10x = 1000 \Rightarrow x = \pm 10 \\ 2\log x - 2\log y = -2 \Rightarrow \log x - \log y = -1 \Rightarrow \log \frac{x}{y} = -1 \Rightarrow \frac{x}{y} = 10^{-1} \Rightarrow y = 10x \end{cases} \\ & \Rightarrow \begin{cases} \cancel{x = -10} \xrightarrow{y=10x} \cancel{y = -100} \quad \cancel{\log(-10)} + \cancel{\log(-100)} = 3 \\ \boxed{x = 10} \xrightarrow{y=10x} \boxed{y = 100} \quad \begin{cases} \log 10 + \log 100 = 3 \checkmark \\ 2\log 10 - 2\log 100 = -2 \checkmark \end{cases} \end{cases} \end{aligned}$$

$$\begin{aligned} 15 \quad & \begin{cases} x - y = 21 \Rightarrow x = 21 + y \\ \log x + \log y = 2 \Rightarrow \log(xy) = 2 \Rightarrow xy = 10^2 \xrightarrow{x=21+y} (21+y) \cdot y = 100 \end{cases} \\ & \Rightarrow y^2 + 21y - 100 = 0 \\ & \Rightarrow \begin{cases} y = -25 \xrightarrow{x=21+y} x = -4 \quad \cancel{\log(-4)} + \dots \\ \boxed{y = 4} \xrightarrow{x=21+y} \boxed{x = 25} \quad \log 21 + \log 4 = 2 \checkmark \end{cases} \end{aligned}$$

$$\begin{aligned} 16 \quad & \begin{cases} \log x + 3\log y = 5 \Rightarrow \log x + 3\log y = 5 \\ \log \frac{x}{y} = 1 \Rightarrow \log x - \log y = 1 \end{cases} \Rightarrow 4\log y = 4 \Rightarrow \log y = 1 \Rightarrow \boxed{y = 10} \\ & \Rightarrow \log \frac{x}{10} = 1 \Rightarrow \frac{x}{10} = 10 \Rightarrow \boxed{x = 100} \Rightarrow \begin{cases} \log 100 + 3\log 10 = 5 \checkmark \\ \log \frac{100}{10} = 1 \checkmark \end{cases} \end{aligned}$$

$$17 \quad \begin{cases} 3x + 2y = 64 \xrightarrow{x=10y} 30y + 2y = 64 \Rightarrow \boxed{y = 2} \Rightarrow \boxed{x = 20} \\ \log x - \log y = 1 \Rightarrow \log \frac{x}{y} = 1 \Rightarrow \frac{x}{y} = 10 \Rightarrow x = 10y \end{cases} \quad \log 20 - \log 2 = 1 \checkmark$$

$$18 \quad \begin{cases} x + y = 30 \xrightarrow{x=9y} 9y + y = 30 \Rightarrow \boxed{y = 3} \xrightarrow{x=9y} \boxed{x = 27} \\ \log_3 x - \log_3 y = 2 \Rightarrow \log_3 \frac{x}{y} = 2 \Rightarrow \frac{x}{y} = 3^2 \Rightarrow x = 9y \end{cases} \quad \log_3 9 - \log_3 3 = 2 \checkmark$$

$$\begin{aligned} 19 \quad & \begin{cases} \log x + 3\log y = 5 \Rightarrow 4\log y = 2 \Rightarrow \log y = \frac{1}{2} \Rightarrow y = 10^{1/2} \Rightarrow \boxed{y = \sqrt{10}} \\ \log x - \log y = 3 \end{cases} \\ & \xrightarrow{\log y = 1/2} \log x - \frac{1}{2} = 3 \Rightarrow \log x = \frac{7}{2} \Rightarrow x = 10^{7/2} \Rightarrow \boxed{x = \sqrt{10^7}} \\ & \begin{cases} \log \sqrt{10^7} + 3\log \sqrt{10} = 5 \checkmark \\ \log \sqrt{10^7} - \log \sqrt{10} = 3 \checkmark \end{cases} \end{aligned}$$

$$\begin{aligned} 20 \quad & \begin{cases} \log x + 5\log y = 7 \Rightarrow \log x + 5\log y = 7 \\ \log \frac{x}{y} = 1 \Rightarrow \log x - \log y = 1 \end{cases} \Rightarrow 6\log y = 6 \xrightarrow{\log y = 1} \boxed{y = 10} \\ & \xrightarrow{\log y = 1} \log x - 1 = 1 \Rightarrow \log x = 2 \Rightarrow \boxed{x = 100} \end{aligned}$$

7.3. Sistemas de ecuaciones logarítmicas

$$\begin{cases} \log 100 + 5 \log 10 = 7 \checkmark \\ \log \frac{100}{10} = 1 \checkmark \end{cases}$$

$$(21) \begin{cases} 2 \log x + \log y = 5 & \Rightarrow 2 \log x + \log y = 5 \\ \log xy = 4 & \Rightarrow \log x + \log y = 4 \end{cases} \Rightarrow \log x = 1 \Rightarrow \boxed{x = 10}$$

$$\xrightarrow{\log x=1} 1 + \log y = 4 \Rightarrow \log y = 3 \Rightarrow y = 10^3 \Rightarrow \boxed{y = 1000}$$

$$\begin{cases} 2 \log 10 + \log 1000 = 5 \checkmark \\ \log (10 \cdot 1000) = 4 \checkmark \end{cases}$$

$$(22) \begin{cases} \log x + \log y^3 = 5 & \Rightarrow \log x + 3 \log y = 5 \\ \log \frac{x^2}{y} = 3 & \Rightarrow 2 \log x - \log y = 3 \end{cases} \Rightarrow 7 \log x = 14 \xrightarrow{\log x=2} \boxed{x = 100}$$

$$\xrightarrow{\log x=2} 2 + 3 \log y = 5 \Rightarrow 3 \log y = 3 \Rightarrow \log y = 1 \Rightarrow \boxed{y = 10}$$

$$\begin{cases} \log 100 + \log 10^3 = 5 \checkmark \\ \log \frac{100^2}{10} = 3 \checkmark \end{cases}$$

$$(23) \begin{cases} y - 4x = 0 & \Rightarrow y = 4x \\ \log x + \log y = 4 & \xrightarrow{y=4x} \log (x \cdot 4x) = 44x^2 = 10^4 \Rightarrow x^2 = 2500 \end{cases}$$

$$\Rightarrow \begin{cases} \cancel{x=50} & \log(\cancel{50}) + \dots \\ \boxed{x=50} & \xrightarrow{y=4x} \boxed{y=200} \quad \log 50 + \log 200 = 4 \checkmark \end{cases}$$

$$(24) \begin{cases} 2 \log x - 3 \log y = 5 & \Rightarrow 2 \log x - 3 \log y = 5 \\ 3 \log x + \log y = 2 & \Rightarrow 9 \log x + 3 \log y = 6 \end{cases} \Rightarrow 11 \log x = 11 \xrightarrow{\log x=1} \boxed{x = 10}$$

$$\xrightarrow{\log x=1} 3 + \log y = 2 \Rightarrow \log y = -1 \Rightarrow y = 10^{-1} \Rightarrow \boxed{y = \frac{1}{10}}$$

$$\begin{cases} 2 \log 10 - 3 \log \frac{1}{10} = 5 \checkmark \\ 3 \log 10 + \log \frac{1}{10} = 2 \checkmark \end{cases}$$

$$(25) \begin{cases} x - 2y = 9 & \Rightarrow x = 2y + 9 \\ \log_3 x - \log_3 y = 1 & \Rightarrow \log_3 \frac{x}{y} = 1 \Rightarrow \frac{x}{y} = 3 \xrightarrow{x=2y+9} 2y + 9 = 3y \Rightarrow \boxed{y = 9} \end{cases}$$

$$\xrightarrow{x=2y+9} \boxed{x = 27} \quad \log_3 27 - \log_3 9 = 1 \checkmark$$

$$(26) \begin{cases} x - y = 3 & \Rightarrow x = 3 + y \\ \log x + \log y = 1 & \Rightarrow \log (xy) = 1 \Rightarrow xy = 10 \xrightarrow{x=3+y} (3+y) \cdot y = 10 \end{cases}$$

$$\Rightarrow y^2 + 3y - 10 = 0 \Rightarrow \begin{cases} y = -5 & \xrightarrow{x=3+y} x = -2 \quad \log(\cancel{-2}) + \dots \\ \boxed{y = 2} & \xrightarrow{x=3+y} \boxed{x = 5} \quad \log 5 + \log 2 = 1 \checkmark \end{cases}$$

$$\begin{aligned}
 27 \quad & \begin{cases} 5^{x+y} = 5^{11} \implies x+y=11 \implies y=11-x \\ \log(x+y) + \log(x-y) = \log 33 \Rightarrow \log[(x+y) \cdot (x-y)] = \log 33 \Rightarrow x^2 - y^2 = 33 \end{cases} \\
 & \xRightarrow{y=11-x} x^2 - (11-x)^2 = 33 \implies x^2 - 121 + x^2 - 22x = 33 \\
 & \implies \boxed{x=4} \xRightarrow{y=11-x} \boxed{y=7} \quad \log(7+4) + \log(7-4) = \log 33 \quad \checkmark
 \end{aligned}$$

HTTPS://APRENDECONMIGOMELON.COM