II INTEGRALES EN FORMA POTENCIAL

Int-1

a) $\int \left(\frac{7}{3}x^2 - 2x + 3 \right) dx$

b) $\int \frac{1 - x^3}{x^2} dx$

c) $\int \frac{2 + 3x^2}{\sqrt{x}} dx$

d) $\int \left(\frac{3x - 5}{2} \right) \cdot \left(x^2 - 1 \right) dx$

e) $\int 6x \cdot \left(3x^2 - 7 \right)^4 dx$

f) $\int x \cdot \sqrt{x^2 + 1}\, dx$

g) $\int \left(\frac{3 - 5x}{2} \right)^3 dx$

h) $\int \frac{3x}{\sqrt{x^2 - 2}} dx$

i) $\int \operatorname{sen}^2 x \cdot \cos x\, dx$

j) $\int \operatorname{tg} x \cdot \sec^2 x\, dx$

k) $\int \frac{x + \sqrt{x}}{x^2} dx$

l) $\boxed{\int_0^1 \frac{x}{\sqrt{4 - x^2}} dx}$

m) $\int \frac{\left(1 + \ln x \right)^2}{x} dx$

n) $\int \sqrt{\left(1 + \cos x \right)^3} \cdot \operatorname{sen} x\, dx$

o) $\int \frac{\ln x}{x} dx$

p) $\int \frac{3x}{\sqrt{1 + 7x^2}} dx$

q) $\int \frac{\left(1 + \sqrt{x} \right)^2}{\sqrt{x}} dx$

Solución:

a) $\int \left(\frac{7}{3}x^2 - 2x + 3 \right) dx = \frac{7}{9}x^3 - x^2 + 3x + k$

b) $\int \frac{1 - x^3}{x^2} dx = \int \frac{dx}{x^2} - \int \frac{x^{\cancel{3}}}{\cancel{x^2}} dx = \int x^{-2}\, dx - \int x\, dx = \frac{x^{-1}}{-1} - \frac{x^2}{2} + k = -\frac{1}{x} - \frac{x^2}{2} + k$

c) $\int \frac{2+3x^2}{\sqrt{x}}\,dx = \int \frac{2}{\sqrt{x}}\,dx + 3\int \frac{x^2}{\sqrt{x}}\,dx = 2\int x^{-1/2}\,dx + 3\int x^{3/2}\,dx = 2\frac{x^{1/2}}{1/2} + 3\frac{x^{5/2}}{5/2} =$

$= 4\sqrt{x} + \frac{6}{5}\sqrt{x^5} + k$

d) $\int \left(\frac{3x-5}{2}\right)\cdot\left(x^2-1\right)dx = \int \frac{3x^3-5x^2-3x+5}{2}\,dx = \frac{1}{2}\left(\frac{3x^4}{4} - \frac{5x^3}{3} - \frac{3x^2}{2} + 5x\right) + k$

e) $\int 6x\cdot\left(3x^2-7\right)^4 dx = \int \underbrace{6x}_{u'}\cdot\underbrace{\left(3x^2-7\right)^4}_{u^4} dx = \frac{1}{5}\cdot\left(3x^2-7\right)^5 + k$

f) $\int x\cdot\sqrt{x^2+1}\,dx = \frac{1}{2}\int \underbrace{2x}_{u'}\cdot\underbrace{\left(x^2+1\right)^{1/2}}_{u^{1/2}} dx = \frac{1}{2}\cdot\frac{\left(x^2+1\right)^{3/2}}{3/2} + k = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\cdot\left(x^2+1\right)^{3/2} + k =$

$= \frac{1}{3}\sqrt{\left(x^2+1\right)^3} + k$

g) $\int \left(\frac{3-5x}{2}\right)^3 dx = \int \frac{1}{8}\cdot(3-5x)^3\,dx = \frac{1}{8}\cdot\frac{1}{-5}\int \underbrace{(-5)}_{u'}\cdot\underbrace{(3-5x)^3}_{u^3} dx = -\frac{1}{40}\cdot\frac{(3-5x)^4}{4} + k =$

$= -\frac{1}{160}\cdot(3-5x)^4 + k$

h) $\int \frac{3x}{\sqrt{x^2-2}}\,dx = 3\int x\cdot\left(x^2-2\right)^{-1/2} dx = \frac{3}{2}\int \underbrace{2x}_{u'}\cdot\underbrace{\left(x^2-2\right)^{-1/2}}_{u^{-1/2}} dx = \frac{3}{\cancel{2}}\cdot\cancel{2}\cdot\left(x^2-2\right)^{1/2} + k =$

$= 3\sqrt{x^2-2} + k$

i) $\int \operatorname{sen}^2 x\cdot\cos x\,dx = \int \underbrace{(\operatorname{sen} x)^2}_{u^2}\cdot\underbrace{\cos x}_{u'}\,dx = \frac{\operatorname{sen}^3 x}{3} + k$

j) $\int \operatorname{tg} x\cdot\sec^2 x\,dx = \int \underbrace{\operatorname{tg} x}_{u}\cdot\underbrace{\sec^2 x}_{u'}\,dx = \frac{1}{2}\operatorname{tg}^2 x + k$

k) $\int \frac{x+\sqrt{x}}{x^2}\,dx = \int \frac{1}{x}\,dx + \int \frac{\sqrt{x}}{x}\,dx = \int \frac{1}{x}\,dx + \int x^{-1/2}\,dx = \ln|x| + \frac{x^{1/2}}{1/2} = \ln|x| + 2\sqrt{x} + k$

$$\boxed{\begin{aligned} &\textbf{l)}\int \frac{x}{\sqrt{4-x^2}}\,dx = \int x\cdot\left(4-x^2\right)^{-1/2} dx = -\frac{1}{2}\cdot\int \underbrace{(-2x)}_{u'}\cdot\underbrace{\left(4-x^2\right)^{-1/2}}_{u^{1/2}} dx = -\frac{1}{\cancel{2}}\cdot\cancel{2}\cdot\left(4-x^2\right)^{1/2} + k = \\ &= -\sqrt{4-x^2} + k \end{aligned}}$$

$$\textbf{m)} \int \frac{(1+\ln x)^2}{x}\,dx = f)\int \underbrace{\frac{1}{x}}_{u'} \cdot \underbrace{(1+\ln x)^2}_{u^2}\,dx = \frac{(1+\ln x)^3}{3} + k$$

$$\textbf{n)} \int \sqrt{(1+\cos x)^3} \cdot \operatorname{sen} x\,dx = \int \underbrace{(1+\cos x)^{3/2}}_{u^{3/2}} \cdot \underbrace{\operatorname{sen} x}_{u'}\,dx = \frac{(1+\cos x)^{5/2}}{5/2} + k = \frac{2}{5}\sqrt{(1+\cos x)^5} + k$$

$$\textbf{o)} \int \frac{\ln x}{x}\,dx = \int \underbrace{\frac{1}{x}}_{u'} \cdot \underbrace{\ln x}_{u}\,dx = \frac{1}{2}(\ln x)^2 + k$$

$$\textbf{p)} \int \frac{3x}{\sqrt{1+7x^2}}\,dx = \int 3x \cdot \left(1+7x^2\right)^{-1/2} dx = \frac{3}{14}\int \underbrace{14x}_{u'} \cdot \underbrace{\left(1+7x^2\right)^{-1/2}}_{u^{-1/2}} dx = \frac{3}{14} \cdot \frac{\left(1+7x^2\right)^{1/2}}{1/2} + k =$$

$$= \frac{6}{14} \cdot \sqrt{1+7x^2} + k$$

$$\textbf{q)} \int \frac{\left(1+\sqrt{x}\right)^2}{\sqrt{x}}\,dx = \int \frac{1}{\sqrt{x}} \cdot \left(1+\sqrt{x}\right)^2 dx = 2\int \underbrace{\frac{1}{2\sqrt{x}}}_{u'} \cdot \underbrace{\left(1+\sqrt{x}\right)^2}_{u^2} dx = \frac{2}{3} \cdot \underbrace{\left(1+\sqrt{x}\right)^3}_{u^2} + k$$

$$\overset{\text{Otra forma}}{=\!=\!=\!=\!=} \int \frac{1+x+2\sqrt{x}}{\sqrt{x}}\,dx = \int \left(\frac{1}{\sqrt{x}} + \sqrt{x} + 2\right) dx = \frac{x^{1/2}}{1/2} + \frac{x^{3/2}}{3/2} + 2x + k =$$

$$= 2\sqrt{x} + \frac{2\sqrt{x^3}}{3} + 2x + k$$

INTEGRALES LOGARÍTMICAS

Int-2 a) $\int \frac{x-1}{x^2-2x+1}dx$ b) $\int \frac{e^x}{1+e^x}dx$

c) $\int \operatorname{cotg} x\,dx$ d) $\int \frac{\operatorname{sen} x+\cos x}{\operatorname{sen} x-\cos x}dx$

e) $\int \frac{2x-3}{x+2}dx$ f) $\int \frac{1}{x\cdot \ln x}dx$

g) $\int \frac{x+\sqrt{x}}{x^2}dx$ h) $\int \frac{1}{\operatorname{sen} x\cdot \cos x}dx$

Solución:

a) $\int \frac{x-1}{x^2-2x+1}dx=\frac{1}{2}\int \frac{2x-2}{x^2-2x+1}dx=\frac{1}{2}\ln\left(x^2-2x+1\right)+k$

b) $\int \frac{e^x}{1+e^x}dx=\ln\left(1+e^x\right)+k$

c) $\int \operatorname{cotg} x\,dx=\int \frac{\cos x}{\operatorname{sen} x}dx=\ln\left(\operatorname{sen} x\right)+k$

d) $\int \frac{\operatorname{sen} x+\cos x}{\operatorname{sen} x-\cos x}dx=\ln\left(\operatorname{sen} x-\cos x\right)+k$

e) $\int \frac{2x-3}{x+2}dx=\left\{\text{Dividimos}\right\}=\int\left(2-\frac{7}{x+2}\right)dx=2\int dx-7\int\left(x+2\right)dx=2x-7\ln\left(x+2\right)+k$

f) $\int \frac{1}{x\cdot \ln x}dx=\int \underbrace{\frac{1}{x}}_{u'}\cdot\underbrace{\frac{1}{\ln x}}_{1/u}dx=\ln\left(\ln x\right)+k$

g) $\int \frac{x+\sqrt{x}}{x^2}\,dx = \int \left(\frac{1}{x} + \frac{\sqrt{x}}{x^2} \right) dx = \int \frac{1}{x}\,dx + \int x^{-3/2}\,dx = \ln|x| + \frac{x^{-1/2}}{-1/2} + k = \ln|x| - \frac{2}{\sqrt{x}} + k$

h) $\int \frac{1}{\operatorname{sen} x \cdot \cos x}\,dx = \int \frac{\operatorname{sen}^2 x + \cos^2 x}{\operatorname{sen} x \cdot \cos x}\,dx = \int \frac{\operatorname{sen}^{\cancel{2}} x}{\cancel{\operatorname{sen} x} \cdot \cos x}\,dx + \int \frac{\cos^{\cancel{2}} x}{\operatorname{sen} x \cdot \cancel{\cos x}}\,dx =$

$= \int \frac{\operatorname{sen} x}{\cos x}\,dx + \int \frac{\cos x}{\operatorname{sen} x}\,dx = -\ln|\cos x| + \ln|\operatorname{sen} x| + k = \ln\left| \frac{\operatorname{sen} x}{\cos x} \right| + k$

INTEGRALES TRIGONOMÉTRICAS

Int-3

a) $\int\left(1-\cos\frac{x}{2}\right)dx$ b) $\int\frac{\operatorname{sen}2x+\cos x}{\cos x}dx$

c) $\int x\cdot\cos\left(x^2+1\right)dx$ d) $\int e^x\cdot\operatorname{sen}e^x\,dx$

e) $\int 3\sec^2\left(x+\pi\right)dx$ f) $\int \operatorname{tg}^2 x\,dx$

g) $\int\frac{1-\cos^2 x}{\operatorname{sen}2x}dx$ h) $\int\frac{1+\operatorname{sen}x}{1-\operatorname{sen}x}dx$

i) $\int\frac{\operatorname{sen}x}{\cos^5 x}dx$ j) $\int\cos^2 x\,dx$

k) $\int\operatorname{sen}^2 x\,dx$ l) $\int\cos^3 x\,dx$

m) $\int\cos^4 x\,dx$ n) $\int\operatorname{sen}\left(2x\right)\cdot\cos x\,dx$

Solución:

a) $\int\left(1-\cos\frac{x}{2}\right)dx=\int dx'-2\int\underbrace{\frac{1}{2}}_{u'}\underbrace{\cos\frac{x}{2}}_{\cos u}dx=x-2\operatorname{sen}\frac{x}{2}+k$

b) $\int\frac{\operatorname{sen}2x+\cos x}{\cos x}dx=\int\frac{2\operatorname{sen}x\cdot\cos x+\cos x}{\cos x}dx=\int\left(2\operatorname{sen}x+1\right)dx=-2\cos x+x+k$

c) $\int x\cdot\cos\left(x^2+1\right)dx=\frac{1}{2}\int\underbrace{2x}_{u'}\cdot\underbrace{\cos\left(x^2+1\right)}_{\cos u}dx=\frac{1}{2}\operatorname{sen}\left(x^2+1\right)+k$

d) $\int e^x\cdot\operatorname{sen}e^x\,dx=-\cos e^x+k$

e) $\int 3\sec^2\left(x+\pi\right)dx=3\operatorname{tg}\left(x+\pi\right)+k$

f) $\int\operatorname{tg}^2 x\,dx=\int\left(1+\operatorname{tg}^2 x-1\right)dx=\int\left(1+\operatorname{tg}^2 x\right)dx-\int dx=\operatorname{tg}x-x+k$

g) $\displaystyle\int \frac{1-\cos^2 x}{\operatorname{sen} 2x}\,dx = \int \frac{\operatorname{sen}^{\cancel{2}} x}{2\,\cancel{\operatorname{sen} x}\cdot \cos x}\,dx = \frac{1}{2}\int \frac{\operatorname{sen} x}{\cos x}\,dx = -\frac{1}{2}\int \frac{-\operatorname{sen} x}{\cos x}\,dx = -\frac{1}{2}\ln\left(\cos x\right) + k$

h) $\displaystyle\int \frac{1+\operatorname{sen} x}{1-\operatorname{sen} x}\,dx = \int \frac{\left(1+\operatorname{sen} x\right)^2}{\left(1-\operatorname{sen} x\right)\cdot\left(1+\operatorname{sen} x\right)}\,dx = \int \frac{1+\operatorname{sen}^2 x + 2\operatorname{sen} x}{1-\operatorname{sen}^2 x}\,dx \overset{1=\operatorname{sen}^2 x+\cos^2 x}{=\!=\!=\!=\!=\!=\!=}$

$\displaystyle = \int \frac{\operatorname{sen}^2 x+\cos^2 x+\operatorname{sen}^2 x+2\operatorname{sen} x}{\cos^2 x}\,dx = \int \frac{2\operatorname{sen}^2 x}{\cos^2 x}\,dx + \int dx + 2\int \frac{\operatorname{sen} x}{\cos^2 x}\,dx =$

$\displaystyle = 2\int \operatorname{tg}^2 x\,dx + \int dx + 2\int \operatorname{sen} x\cdot\cos^{-2} x\,dx = 2\int\left(1+\operatorname{tg}^2 x - 1\right)dx + \int dx +$

$\displaystyle +2\int \underbrace{\operatorname{sen} x}_{u}\cdot\underbrace{\cos^{-2}}_{u^{-2}} x\,dx = 2\left(\operatorname{tg} x - x\right) + x - 2\cos^{-1} x + k = 2\operatorname{tg} x - x - \frac{2}{\cos x} + k$

i) $\displaystyle\int \frac{\operatorname{sen} x}{\cos^5 x}\,dx = \int \underbrace{\operatorname{sen} x}_{u'}\cdot\underbrace{\cos^{-5} x}_{u^{-5}}\,dx = -\frac{1}{4}\cos^{-4} x + k = -\frac{1}{4\cos^4 x} + k$

j) $\displaystyle\int \cos^2 x\,dx = \left\{\begin{array}{l} \cos 2x = \cos^2 x - \operatorname{sen}^2 x = \cos^2 x - \left(1-\cos^2 x\right) = 2\cos^2 x - 1 \quad \rightarrow \\ \rightarrow \quad \cos^2 x = \dfrac{1+\cos 2x}{2} = \dfrac{1}{2} + \dfrac{1}{2}\cos 2x \end{array}\right\} =$

$\displaystyle = \int\left(\frac{1}{2}+\frac{1}{2}\cos 2x\right)dx = \int \frac{1}{2}\,dx + \frac{1}{2}\cdot\frac{1}{2}\int \underbrace{2}_{u'}\cdot\underbrace{\cos 2x}_{\cos u}\,dx = \frac{x}{2} + \frac{1}{4}\operatorname{sen} 2x + k$

k) $\displaystyle\int \operatorname{sen}^2 x\,dx \overset{\operatorname{sen}^2 x+\cos^2 x=1}{=\!=\!=\!=} \int\left(1-\cos^2 x\right)dx = x - \int \cos^2 x\,dx \overset{\text{del ejercicio anterior}}{=\!=\!=\!=} x - \left(\frac{x}{2} + \frac{1}{4}\operatorname{sen} 2x + k\right) =$

$\displaystyle = \frac{x}{2} - \frac{1}{4}\operatorname{sen} 2x + k$

l) $\displaystyle\int \cos^3 x\,dx = \int \cos x\cdot\cos^2 x\,dx = \int \cos x\cdot\left(1-\operatorname{sen}^2 x\right)dx = \int \cos x\,dx - \int \underbrace{\cos x}_{u'}\cdot\underbrace{\operatorname{sen}^2 x}_{u^2}\,dx =$

$\displaystyle = \operatorname{sen} x - \frac{1}{3}\operatorname{sen}^3 x + k$

m) $\int \cos^4 x\,dx = \left\{ \begin{array}{l} \cos 2x = \cos^2 x - \operatorname{sen}^2 x = \cos^2 x - \left(1-\cos^2 x\right) = 2\cos^2 x - 1 \quad \rightarrow \\ \rightarrow \quad \cos^2 x = \dfrac{1+\cos 2x}{2} = \dfrac{1}{2} + \dfrac{1}{2}\cos 2x \end{array} \right\} =$

$$= \int \left(\frac{1}{2}+\frac{1}{2}\cos 2x\right)^2 dx = \int \left(\frac{1}{4}+\frac{1}{2}\cos 2x+\frac{1}{4}\cos^2 2x\right)dx = \frac{x}{4}+\frac{1}{4}\operatorname{sen} 2x + \int \frac{1}{4}\cos^2 2x\,dx =$$

$$\overset{\cos^2 2x = \frac{1}{2}+\frac{1}{2}\cos 4x}{=\!=\!=\!=\!=} \frac{x}{4}+\frac{1}{4}\operatorname{sen} 2x + \frac{1}{4}\int \left(\frac{1}{2}+\frac{1}{2}\cos 4x\right)dx = \frac{x}{4}+\frac{1}{4}\operatorname{sen} 2x+\frac{x}{8}+\frac{1}{32}\operatorname{sen} 4x + k =$$

$$= \frac{3x}{8}+\frac{1}{4}\operatorname{sen} 2x+\frac{1}{32}\operatorname{sen} 4x + k$$

n) $\int \operatorname{sen}(2x)\cdot\cos x\,dx = \int 2\cdot \operatorname{sen} x\cdot\cos x\cdot\cos x\,dx = \int 2\cdot\operatorname{sen} x\cdot\cos^2 x\,dx =$

$$= -2\cdot\int \underbrace{-\operatorname{sen} x}_{u'}\cdot\underbrace{\cos^2 x}_{u^2}\,dx = -\frac{2}{3}\cos^3 x + k$$

INTEGRALES DE FORMA ARCO

Int-4

a) $\displaystyle\int \frac{x^2}{\sqrt{1-x^6}}\,dx$

b) $\displaystyle\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

c) $\displaystyle\int \frac{1}{1+4x^2}\,dx$

d) $\displaystyle\int \frac{1}{9+x^2}\,dx$

e) $\displaystyle\int \frac{1}{4+5x^2}\,dx$

f) $\displaystyle\int \frac{1}{x^2+2x+2}\,dx$

g) $\displaystyle\int \frac{x^2}{1+x^2}\,dx$

h) $\displaystyle\int \frac{x^4}{x^2+1}\,dx$

i) $\displaystyle\int \frac{1}{\sqrt{4-3x^2}}\,dx$

j) $\displaystyle\int \frac{x+3}{\sqrt{9-x^2}}\,dx$

k) $\displaystyle\int \frac{1}{x^2+x+1}\,dx$

l) $\displaystyle\int \frac{1}{4+7x^2}\,dx$

Solución:

a) $$\int \frac{x^2}{\sqrt{1-x^6}}\,dx = \int \frac{x^2}{\sqrt{1-\left(x^3\right)^2}}\,dx = \frac{1}{3}\cdot\int \underbrace{\frac{3x^2}{\sqrt{1-\left(x^3\right)^2}}}_{\frac{u'}{\sqrt{1-u^2}}}\,dx = \frac{1}{3}\cdot \operatorname{arcsen} x^3 + k$$

b) $$\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = \int \underbrace{\frac{e^x}{\sqrt{1-\left(e^x\right)^2}}}_{\frac{u'}{\sqrt{1-u^2}}}\,dx = \operatorname{arcsen} e^x + k$$

$$\textbf{c)}\int\frac{1}{1+4x^2}\,dx=\int\frac{1}{1+(2x)^2}\,dx=\frac{1}{2}\cdot\int\underbrace{\frac{2}{1+(2x)^2}}_{\frac{u'}{1+u^2}}\,dx=\frac{1}{2}\cdot\operatorname{arctg}2x+k$$

$$\textbf{d)}\int\frac{1}{9+x^2}\,dx=\int\frac{1/9}{9\cdot\left(1+x^2/9\right)}\,dx=\frac{1}{9}\cdot\int\frac{1}{1+(x/3)^2}\,dx=\frac{1}{9}\cdot3\cdot\int\underbrace{\frac{1/3}{1+(x/3)^2}}_{\frac{u'}{1+u^2}}\,dx=\frac{1}{3}\cdot\operatorname{arctg}(x/3)+k$$

$$\textbf{e)}\int\frac{1}{4+5x^2}\,dx=\int\frac{1/4}{1+5/4\,x^2}\,dx=\frac{1}{4}\cdot\int\frac{1}{1+\left(\sqrt{5}/2\,x\right)^2}\,dx=\frac{1}{4}\cdot\frac{2}{\sqrt{5}}\cdot\int\underbrace{\frac{\sqrt{5}/2}{1+\left(\sqrt{5}/2\,x\right)^2}}_{\frac{u'}{1+u^2}}\,dx=$$

$$=\frac{1}{2\sqrt{5}}\cdot\operatorname{arctg}\left(\frac{\sqrt{5}x}{2}\right)+k$$

$$\textbf{f)}\int\frac{1}{x^2+2x+2}\,dx=\int\frac{1}{x^2+2x+1+1}\,dx=\left\{(x+1)^2=x^2+2x+1\right\}=\int\underbrace{\frac{1}{1+(x+1)^2}}_{\frac{u'}{1+u^2}}\,dx=$$

$$=\operatorname{arctg}(x+1)+k$$

$$\textbf{g)}\int\frac{x^2}{1+x^2}\,dx\overset{\text{Dividimos}}{=\!=\!=\!=}\int\left(1-\frac{1}{1+x^2}=\right)dx=\int dx-\int\frac{1}{1+x^2}\,dx=x-\operatorname{arctg}x+k$$

$$\textbf{h)}\int\frac{x^4}{x^2+1}\,dx\overset{\text{Dividimos}}{=\!=\!=\!=}\int\left(x^2-1+\frac{1}{x^2+1}\right)dx=\frac{x^3}{3}-x+\operatorname{arctg}x+k$$

$$\textbf{i)}\int\frac{1}{\sqrt{4-3x^2}}\,dx=\int\frac{1}{\sqrt{4\cdot\left(1-3x^2/4\right)}}\,dx=\frac{1}{2}\cdot\int\frac{1}{\sqrt{1-\left(\sqrt{3}x/2\right)^2}}\,dx=\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{3}}\cdot\int\underbrace{\frac{\sqrt{3}/2}{\sqrt{1-\left(\sqrt{3}x/2\right)^2}}}_{\frac{u'}{\sqrt{1-u^2}}}\,dx=$$

$$=\frac{1}{\sqrt{3}}\cdot\operatorname{arcsen}\left(\frac{\sqrt{3}x}{2}\right)+k$$

j) $\int \frac{x+3}{\sqrt{9-x^2}}dx = \int \frac{x}{\sqrt{9-x^2}}dx + \int \frac{3}{\sqrt{9-x^2}}dx = \int x\cdot\left(9-x^2\right)^{-1/2}dx + \int \frac{3}{\sqrt{9\cdot\left(1-x^2/9\right)}}dx =$

$= -\frac{1}{2}\int \underbrace{(-2x)}_{u'}\cdot\underbrace{\left(9-x^2\right)^{-1/2}}_{u^{-1/2}}dx + \int \frac{3}{\sqrt{9\cdot\left(1-x^2/9\right)}}dx = -\frac{1}{2}\int \underbrace{(-2x)}_{u'}\cdot\underbrace{\left(9-x^2\right)^{-1/2}}_{u^{-1/2}}dx +$

$+\int \frac{\cancel{3}}{\cancel{3}\cdot\sqrt{1-(x/3)^2}}dx = -\frac{1}{2}\int \underbrace{(-2x)}_{u'}\cdot\underbrace{\left(9-x^2\right)^{-1/2}}_{u^{-1/2}}dx + 3\cdot\int \underbrace{\frac{1/3}{\sqrt{1-(x/3)^2}}}_{\frac{u'}{\sqrt{1-u^2}}}dx =$

$= -\frac{1}{\cancel{2}}\cdot\cancel{2}\cdot\left(9-x^2\right)^{1/2} + 3\cdot\operatorname{arcsen}(x/3) + k = -\sqrt{9-x^2} + 3\cdot\operatorname{arcsen}(x/3) + k$

k) $\int \frac{1}{x^2+x+1}dx = \left\{\begin{array}{l}(x+1/2)^2 = x^2+x+1/4\\ x^2+x+1 = (x+1/2)^2+3/4\end{array}\right\} = \int \frac{1}{(x+1/2)^2+3/4}dx =$

$= \int \frac{1}{\frac{3}{4}\cdot\left[\frac{4}{3}(x+1/2)^2+1\right]}dx = \frac{4}{3}\int \frac{1}{\left(\frac{2x+1}{\sqrt{3}}\right)^2+1}dx =$

$= \frac{4}{3}\cdot\frac{\sqrt{3}}{2}\int \underbrace{\frac{2/\sqrt{3}}{1+\left(\frac{2x+1}{\sqrt{3}}\right)^2}}_{\frac{u'}{1+u^2}}dx = \frac{2\sqrt{3}}{3}\operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right) + k$

l) $\int \frac{1}{4+7x^2}dx = \int \frac{1}{4\cdot\left(1+7x^2/4\right)}dx = \int \frac{1/4}{1+\left(\frac{x\sqrt{7}}{2}\right)^2}dx = \frac{1}{2\sqrt{7}}\int \frac{\sqrt{7}/2}{1+\left(\frac{x\sqrt{7}}{2}\right)^2}dx = \operatorname{arctg}\left(\frac{x\sqrt{7}}{2}\right) + k$

MÉTODO DE SUSTITUCIÓN

Int-5 a) $\int \operatorname{sen}^4 x \cdot \cos x\,dx$ b) $\int e^{3x+1}\,dx$ c) $\int e^{x^2-5x}\cdot(2x-5)\,dx$

d) $\int \frac{x}{\sqrt{1-x^4}}\,dx$ e) $\int \operatorname{tg} x\,dx$ f) $\int \frac{dx}{(1+x)\cdot\sqrt{x}}$

g) $\int 2^{\operatorname{sen} x}\cdot\cos x\,dx$ h) $\int \frac{dx}{\cos^2 x\cdot\sqrt[3]{1+\operatorname{tg} x}}$ i) $\int \frac{dx}{x\cdot\sqrt{1-\ln x}}$

j) $\int \frac{x^2}{\sqrt[3]{1+2x}}\,dx$ k) $\frac{x}{1+\sqrt{x}}\,dx$ l) $\int\sqrt{e^x-1}\,dx$

m) $\int \frac{x+1}{x^2+2x+7}\,dx$ n) $\int \frac{1}{\sqrt{x}\cdot(1+x)}\,dx$

Solución:

a) $\int \operatorname{sen}^4 x\cdot\cos x\,dx = \left\{\begin{array}{l} t=\operatorname{sen} x \\ dt=\cos x\,dx \end{array}\right\} = \int t^4\,dt = \frac{t^5}{5} = \frac{1}{5}\operatorname{sen}^5 x + k$

b) $\int e^{3x+1}\,dx = \left\{\begin{array}{l} t=3x+1 \\ dt=3\,dx \end{array}\right\} = \frac{1}{3}\int e^t\,dt = \frac{1}{3}e^t + k = \frac{1}{3}e^{3x+1}+k$

c) $\int e^{x^2-5x}\cdot(2x-5)\,dx = \left\{\begin{array}{l} t=x^2+5x \\ dt=(2x+5)\,dx \end{array}\right\} = \int e^t\,dt = e^t + k = e^{x^2+5x}+k$

d) $\int \frac{x}{\sqrt{1-x^4}}\,dx = \left\{\begin{array}{l} t=x^2 \\ dt=2x\,dx \end{array}\right\} = \int \frac{1/2}{\sqrt{1-t^2}}\,dt = \frac{1}{2}\int\frac{dt}{\sqrt{1-t^2}} = \frac{1}{2}arc\operatorname{sen} t = \frac{1}{2}arc\operatorname{sen} x^2 + k$

e) $\int \operatorname{tg} x\,dx = \int\frac{\operatorname{sen} x}{\cos x}\,dx = \left\{\begin{array}{l} t=\cos x \\ dt=-\operatorname{sen} x\,dx \end{array}\right\} = -\int\frac{dt}{t} = -\ln t + k = -\ln(\cos x)+k$

$$\textbf{f)}\int \frac{dx}{(1+x)\cdot\sqrt{x}} = \left\{\begin{array}{l} t=\sqrt{x} \\ dt = \dfrac{1}{2\sqrt{x}}\,dx \end{array}\right\} = \int \frac{2\cancel{\sqrt{x}}\;dt}{(1+t^2)\cdot\cancel{\sqrt{x}}} = 2\int \frac{1}{1+t^2}\,dt = 2\,\mathrm{arctg}\,t + k = 2\,\mathrm{arctg}\,\sqrt{x} + k$$

$$\textbf{g)}\int 2^{\mathrm{sen}\,x}\cdot\cos x\,dx = \left\{\begin{array}{l} t=\mathrm{sen}\,x \\ dt=\cos x\,dx \end{array}\right\} = \int 2^t\,dt = \frac{1}{\ln 2}2^t + k = \frac{2^{\mathrm{sen}\,x}}{\ln 2}+k$$

$$\textbf{h)}\int \frac{dx}{\cos^2 x\cdot\sqrt[3]{1+\mathrm{tg}\,x}} = \left\{\begin{array}{l} t=1+\mathrm{tg}\,x \\ dt=\sec^2 x\,dx \end{array}\right\} = \int\frac{dt}{\sqrt[3]{t}} = \int t^{-1/3}\,dt = \frac{t^{2/3}}{2/3}+k = \frac{3}{2}\cdot\sqrt[3]{(1+\mathrm{tg}\,x)^2}+k$$

$$\textbf{i)}\int \frac{dx}{x\cdot\sqrt{1-\ln x}} = \left\{\begin{array}{l} t=1-\ln x \\ dt=-\dfrac{1}{x}\,dx \end{array}\right\} = \int -\frac{dt}{\sqrt{t}} = \int -t^{-1/2}\,dt = -\frac{t^{1/2}}{1/2}+k = -2\sqrt{t}+k = -2\sqrt{1-\ln x}+k$$

$$\textbf{j)}\int \frac{x^2}{\sqrt[3]{1+2x}}\,dx = \left\{\begin{array}{lll} t^3=1+2x & \to & x=\dfrac{t^3-1}{2} \\ 3t^2\,dt=2dx & \to & dx=\dfrac{3t^2}{2}\,dt \end{array}\right\} = \int\left(\frac{t^3-1}{2}\right)^2\cdot\frac{1}{t}\cdot\frac{3t^2}{2}\,dt = \frac{3}{8}\int\left(t^7-2t^4+t\right)dt =$$

$$= \frac{3}{8}\left(\frac{t^8}{8}-\frac{2t^5}{5}+\frac{t^2}{2}\right)+k = \frac{3}{8}\left(\frac{t^8}{8}-\frac{2t^5}{5}+\frac{t^2}{2}\right)+k = \frac{3}{8}t^2\cdot\left(\frac{t^6}{8}-\frac{2t^3}{5}+\frac{1}{2}\right)+k =$$

$$= \frac{3}{8}\sqrt[3]{(1+2x)^2}\cdot\left(\frac{(1+2x)^2}{8}-\frac{2(1+2x)}{5}+\frac{1}{2}\right)+k$$

$$\textbf{k)}\int \frac{x}{1+\sqrt{x}}\,dx = \left\{\begin{array}{l} x=t^2 \\ dx=2t\,dt \end{array}\right\} = \int\frac{t^2}{1+t}\cdot 2t\,dt = \int\frac{2t^3}{1+t}\,dt = 2\int\left(t^2-t+1-\frac{1}{1+t}\right)dt =$$

$$= 2\left[\frac{t^3}{3}-\frac{t^2}{2}+t-\ln|1+t|\right]+k = 2\left[\frac{\sqrt{x^3}}{3}-\frac{x}{2}+\sqrt{x}-\ln\left(1+\sqrt{x}\right)\right]+k$$

$$\textbf{l)}\int\sqrt{e^x-1}\,dx = \left\{\begin{array}{lll} x^2-1=t^2 & \to & x^2=1+t^2 \\ x^2\,dx=2t\,dt & \to & dx=\dfrac{2t\,dt}{t^2+1} \end{array}\right\} = \int t\cdot\frac{2t\,dt}{t^2+1} = 2\int\frac{t^2}{t^2+1}\,dt =$$

$$= 2\int\left(1-\frac{1}{t^2+1}\right)dt = 2\left(t-\mathrm{arctg}\,t\right)+k = 2\left(\sqrt{e^2-1}-\mathrm{arctg}\,\sqrt{e^x-1}\right)+k$$

$$\textbf{m)}\int\frac{x+1}{x^2+2x+7}\,dx = \left\{\begin{array}{lll} u=x^2+2x+7 & & \\ du=2x+2 & \to & x+1=\dfrac{1}{2}\,du \end{array}\right\} = \frac{1}{2}\int\frac{du}{u} = \frac{1}{2}\ln u = \frac{1}{2}\ln\left(x^2+2x+7\right)+k$$

n) $\int \frac{1}{\sqrt{x}\cdot(1+x)}dx = \left\{\begin{matrix} t=\sqrt{x} \quad \rightarrow \quad t^2=x \\ 2tdt=dx \end{matrix}\right\} = \int \frac{2\cancel{t}}{\cancel{t}\cdot(1+t^2)}dt = 2\int \frac{1}{(1+t^2)}dt = 2\cdot \text{arctg}\, t + k =$

$= 2\cdot \text{arctg}\left(\sqrt{x}\right) + k$

INTEGRALES RACIONALES

Int-6

a) $\displaystyle\int \frac{x^2+x+1}{x}\,dx$ b) $\displaystyle\int \frac{3x^2-7x+4}{2x-3}\,dx$

c) $\displaystyle\int \frac{x^3+4x^2-10x+7}{x^3-7x-6}$ d) $\displaystyle\int \frac{5x-3}{x^3-x}\,dx$

e) $\displaystyle\int \frac{2x+5}{(x+3)^3}\,dx$ f) $\displaystyle\int \frac{x^2-2x+6}{(x-1)^3}\,dx$

g) $\displaystyle\int \frac{6x^5-7x^4-5x^3+x^2-5x+2}{x^6-2x^5+2x^3-x^2}\,dx$ h) $\displaystyle\int \frac{3x^2-5x+1}{x-4}\,dx$

i) $\displaystyle\int \frac{3x^2-5x+1}{2x+1}\,dx$ j) $\displaystyle\int \frac{x^3+22x^2-12x+8}{x^4-4x^2}\,dx$

k) $\displaystyle\int \frac{x^3-4x^2+4x}{x^4-2x^3-4x^2+8x}\,dx$ l) $\displaystyle\int \frac{x^4+2x-6}{x^3+x^2-2x}\,dx$

m) $\displaystyle\int \frac{5x^2}{x^3-3x^2+3x-1}\,dx$ n) $\displaystyle\int \frac{x^3}{x^2-1}\,dx$

o) $\displaystyle\int \frac{3x-2}{x^2-4}\,dx$

Solución:

a) $\displaystyle\int \frac{x^2+x+1}{x}\,dx=\int\left(x+1+\frac{1}{x}\right)dx=\frac{x^2}{x}+x+\ln x+k$

b) $\displaystyle I=\int \frac{3x^2-7x+4}{2x-3}\,dx$ Si grado numerador $\geq$ grado denominador $\rightarrow$ dividimos

$$\begin{array}{rrr|l}
\cancel{3x^2} & -7x & +4 & 2x-3 \\ \cline{4-4}
\cancel{-3x^2} & +9x/2 & & 3x/2-5/4 \\ \hline
 & \cancel{-5x/2} & +4 & \\
 & \cancel{5x/2} & -15/4 & \\ \cline{2-3}
 & & 1/4 &
\end{array}$$

$$\frac{3x^2-7x+4}{2x-3}=\frac{3x}{2}-\frac{5}{4}+\frac{1/4}{2x-3} \rightarrow \int\frac{3x^2-7x+4}{2x-3}dx=\int\left(\frac{3x}{2}-\frac{5}{4}+\frac{1/4}{2x-3}\right)dx=$$

$$=\int\frac{3x}{2}dx-\int\frac{5}{4}dx+\int\frac{dx}{8x-12} \rightarrow \boxed{I=\frac{3x^2}{4}-\frac{5x}{4}+\frac{1}{8}\ln|8x-12|+k}$$

c) $I=\int\frac{x^3+4x^2-10x+7}{x^3-7x-6}dx$

Como $grado\ numerador \geq grado\ denominador$, hacemos la división y posteriormente factorizamos el denominador para determinar el número y el tipo de raices del mismo

$$\begin{array}{rrrr|l}
\cancel{x^3} & +4x^2 & -10x & 7 & x^3-7x-6 \\ \cline{5-5}
\cancel{-x^3} & & +7x & +6 & 1 \\ \cline{1-4}
 & +4x^2 & -3x & +13 &
\end{array}$$

$$\begin{array}{r|rrrr}
 & 1 & 0 & -7 & -6 \\
-1 & & -1 & 1 & 6 \\ \hline
 & 1 & -1 & -6 & \boxed{0} \\
3 & & 3 & 6 & \\ \hline
 & 1 & 2 & \boxed{0} &
\end{array}$$

$$\frac{4x^2-3x+13}{x^3-7x-6}=\frac{A}{x+1}+\frac{B}{x-3}+\frac{C}{x+2}=$$

$$=\frac{A\cdot(x-3)\cdot(x+2)+B\cdot(x+1)\cdot(x+2)+C\cdot(x+1)\cdot(x-3)}{(x+1)\cdot(x-3)\cdot(x+2)} \rightarrow$$

$$\rightarrow \left\langle \begin{array}{lll} x=-1 & \rightarrow & 20=-4A \\ x=3 & \rightarrow & 40=20B \\ x=-2 & \rightarrow & 35=5C \end{array}\right. \rightarrow \boxed{\begin{array}{l} A=-5 \\ B=2 \\ C=7 \end{array}}$$

$$\rightarrow \quad I=\int\left(1+\frac{4x^2-3x+13}{x^3-7x-6}\right)dx=\int\left(1+\frac{5}{x+1}+\frac{2}{x-3}+\frac{7}{x+2}\right)dx \rightarrow$$

$$\boxed{I=x-5\ln|x+1|+2\ln|x-3|+7\ln|x+2|+k}$$

d) $I = \int \frac{5x-3}{x^3-x}\,dx$

Como *grado numerador* < *grado denominador* factorizamos el denominador:

$$x^3 - x = x\cdot(x^2-1) = x\cdot(x+1)\cdot(x-1) \rightarrow \frac{5x-3}{x^3-x} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} =$$

$$= \frac{A\cdot(x+1)\cdot(x-1) + Bx\cdot(x-1) + Cx\cdot(x+1)}{x\cdot(x+1)\cdot(x-1)} \left\langle \begin{matrix} x=0 & \rightarrow & -3=-A \\ x=-1 & \rightarrow & -8=2B \\ x=1 & \rightarrow & 2=2C \end{matrix} \right. \rightarrow \boxed{\begin{matrix} A=3 \\ B=-4 \\ C=1 \end{matrix}}$$

$$\rightarrow \quad I = \textbf{d)}\, I = \int \frac{5x-3}{x^3-x}\,dx = \int \left(\frac{3}{x} - \frac{4}{x+1} + \frac{1}{x-1} \right) dx \rightarrow$$

$$\boxed{I = 3\ln|x| - 4\ln|x+1| + \ln|x-1| + k}$$

e) $I = \int \frac{2x+5}{(x+3)^3}\,dx$

Grado numerador < *grado denominador*, y como el denominador ya está factorizado:

$$\frac{2x+5}{(x+3)^3} = \frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{(x+3)^3} = \frac{A\cdot(x+3)^2 + B\cdot(x+3) + C}{(x+3)^3} \rightarrow$$

$$\rightarrow \left\langle \begin{matrix} (x^2) & \rightarrow & 0 = A & \rightarrow \\ (x) & \rightarrow & 2 = 6A + B & \rightarrow \\ T.I. & \rightarrow & 5 = 9A + 3B + C & \rightarrow \end{matrix} \right. \boxed{\begin{matrix} A=0 \\ B=2 \\ C=-1 \end{matrix}}$$

$$I = \int \frac{2x+5}{(x+3)^3}\,dx = \int \left(\frac{2}{(x+3)^2} - \frac{1}{(x+3)^3} \right) dx = \int \left[2\cdot(x+3)^{-2} - (x+3)^{-3} \right] dx =$$

$$= 2\cdot\frac{(x+3)^{-1}}{-1} - \frac{(x+3)^{-2}}{-2} + k \rightarrow \boxed{I = \frac{-2}{x+3} + \frac{1}{2\cdot(x+3)^2} + k}$$

f) $\int \frac{x^2-2x+6}{(x-1)^3}\,dx$

Grado numerador < *grado denominador*, y como el denominador ya está factorizado:

$$\frac{x^2-2x+6}{(x-1)^3}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}=\frac{A\cdot(x-1)^2+B\cdot(x-1)+C}{(x-1)^3}\rightarrow$$

$$\rightarrow\left\langle\begin{array}{lll}(x^2) & \rightarrow 1=A & \rightarrow \\ (x) & \rightarrow -2=-2A+B & \rightarrow \\ T.I. & \rightarrow 6=A-B+C & \rightarrow\end{array}\right. \boxed{\begin{array}{l}A=1\\ B=0\\ C=5\end{array}}$$

$$I=\int\frac{2x+5}{(x+3)^3}dx=\int\left(\frac{1}{x-1}+\frac{5}{(x-1)^3}\right)dx=\int\left[\frac{1}{x-1}+5\cdot(x-1)^{-3}\right]dx=$$

$$=\ln|x-1|+5\cdot\frac{(x-1)^{-2}}{-2}+k\rightarrow \boxed{I=\ln|x-1|-\frac{5}{2\cdot(x-1)^2}+k}$$

g) $I=\int\frac{6x^5-7x^4-5x^3+x^2-5x+2}{x^6-2x^5+2x^3-x^2}dx$

Como *grado numerador* < *grado denominador*, solo tenemosque factorizar éste. Para ello saco x como factor común y luego hago Rufinni.

$$x^6-2x^5+2x^3-x^2=x^2\cdot(x^4-2x^3+2x-1)=x^2\cdot(x-1)^3\cdot(x+1)$$

	1	-2	0	2	-1
1		1	-1	-1	1
	1	-1	-1	1	0
1		1	0	-1	
	1	0	-1	0	
1		1	1		
	1	1	0		

$$\frac{6x^5-7x^4-5x^3+x^2-5x+2}{x^6-2x^5+2x^3-x^2}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}+\frac{D}{(x+1)}+\frac{E}{x}+\frac{F}{x^2}=$$

$$=\frac{A\cdot x^2\cdot(x+1)\cdot(x-1)^2+B\cdot x^2\cdot(x+1)\cdot(x-1)+C\cdot x^2\cdot(x+1)+D\cdot x^2\cdot(x-1)^3}{x^2\cdot(x-1)^3\cdot(x+1)}+$$

$$+\frac{E\cdot x\cdot(x+1)\cdot(x-1)^3+F\cdot(x+1)\cdot(x-1)^3}{x^2\cdot(x-1)^3\cdot(x+1)}\rightarrow$$

$$\rightarrow \begin{cases} x=1 \rightarrow -8=2C & \longrightarrow & C=-4 \\ x=-1 \rightarrow 0=-8D & \longrightarrow & D=0 \\ x=0 \rightarrow 2=-F & \longrightarrow & F=-2 \\ (x) \rightarrow -5=-E & \longrightarrow & E=5 \\ (x^4) \rightarrow -7=-A+B-3D-2E+F & \xrightarrow{-7=-1+B-0-10-2} & B=6 \\ (x^5) \rightarrow 6=A+D+E & \xrightarrow{6=A+0+5} & A=1 \end{cases}$$

$$\frac{6x^5-7x^4-5x^3+x^2-5x+2}{x^6-2x^5+2x^3-x^2}=\frac{1}{x-1}+\frac{6}{(x-1)^2}-\frac{4}{(x-1)^3}+\frac{5}{x}-\frac{2}{x^2}$$

$$\rightarrow \int\frac{6x^5-7x^4-5x^3+x^2-5x+2}{x^6-2x^5+2x^3-x^2}dx=\int\frac{dx}{x-1}+6\int\frac{dx}{(x-1)^2}-4\int\frac{dx}{(x-1)^3}+5\int\frac{dx}{x}-2\int\frac{dx}{x^2} \rightarrow$$

$$\rightarrow \boxed{I=\ln|x-1|-\frac{6}{x-1}+\frac{2}{(x-1)^2}+5\ln|x|+\frac{2}{x}+k}$$

Ha desaparecido la fracción correspondiente a la raíz $x=-1$ porque también existía en el numerador. Si nos hubiesemos dado cuenta a tiempo podríamos haber simplificado previamente la fracción por $x+1$

h) $I=\int\frac{3x^2-5x+1}{x-4}dx$

Como *grado numerador* > *grado denominador*, dividimos:

$$\begin{array}{rrr|l} \cancel{3x^2} & -5x & +1 & x-4 \\ \cancel{-3x^2} & +12x & & 3x+7 \\ \hline & \cancel{7x} & +1 & \\ & \cancel{-7x} & +28 & \\ \hline & & +29 & \end{array}$$

$$\frac{3x^2-5x+1}{x-4}=3x+7+\frac{28}{x-4} \rightarrow \int\frac{3x^2-5x+1}{x-4}dx=\int\left(3x+7+\frac{29}{x-4}\right)dx \rightarrow$$

$$\rightarrow \boxed{I=\frac{3x^2}{2}+7x+29\ln|x-4|+k}$$

i) $I\int\frac{3x^2-5x+1}{2x+1}dx$

Como *grado numerador* ≥ *grado denominador*, dividimos:

$$\begin{array}{rrr|l} \cancel{3x^2} & -5x & +1 & 2x+1 \\ \cancel{-3x^2} & -\frac{3}{2}x & & \hline \frac{3}{2}x-\frac{13}{4} \\ \hline & \cancel{-\frac{13}{2}x} & +1 & \\ & \cancel{\frac{13}{2}x} & +\frac{13}{4} & \\ \hline & & \frac{17}{4} & \end{array}$$

$$\frac{3x^2-5x+1}{2x+1}=\tfrac{3}{2}x-\tfrac{13}{4}+\frac{\frac{17}{4}}{2x+1}\quad\rightarrow\quad\int\frac{3x^2-5x+1}{2x+1}dx=\int\left(\tfrac{3}{2}x-\tfrac{13}{4}+\frac{\frac{17}{4}}{2x+1}\right)dx=$$

$$=\frac{3}{4}x^2-\frac{13}{4}x+\frac{17}{4}\int\frac{dx}{2x+1}=\frac{3}{4}x^2-\frac{13}{4}x+\frac{17}{4}\cdot\frac{1}{2}\int\frac{2}{2x+1}dx\quad\rightarrow$$

$$I=\frac{3}{4}x^2-\frac{13}{4}x+\frac{17}{8}\cdot\ln|2x+1|\quad\rightarrow\quad\boxed{I=\frac{3}{4}x^2-\frac{13}{4}x+\frac{17}{8}\cdot\ln|2x+1|+k}$$

j) $\int\frac{x^3+22x^2-12x+8}{x^4-4x^2}dx$

Como *grado numerador* < *grado denominador*, factorizamos el denominador:

$$x^4-4x^2=x^2\cdot\left(x^2-4\right)=x^2\cdot(x-2)\cdot(x+2)\quad\rightarrow$$

$$\rightarrow\quad\frac{x^3+22x^2-12x+8}{x^4-4x^2}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-2}+\frac{D}{x+2}=$$

$$=\frac{Ax\cdot(x-2)\cdot(x+2)+B\cdot(x-2)\cdot(x+2)+Cx^2\cdot(x+2)+Dx^2\cdot(x-2)}{x^2\cdot(x-2)\cdot(x+2)}\quad\rightarrow$$

$$\rightarrow\left\langle\begin{array}{lcll} x=2 & \rightarrow & 80=16C & \rightarrow \\ x=-2 & \rightarrow & 112=-16D & \rightarrow \\ x=0 & \rightarrow & 8=-4B & \rightarrow \\ \left(x^3\right) & \rightarrow & 1=A+C+D & \xrightarrow{1=A+5-7} \end{array}\right.\quad\boxed{\begin{array}{l} C=5 \\ D=-7 \\ B=-2 \\ A=3 \end{array}}$$

$$\int\frac{x^3+22x^2-12x+8}{x^4-4x^2}dx=\int\left(\frac{3}{x}-\frac{2}{x^2}+\frac{5}{x-2}-\frac{7}{x+2}\right)dx\quad\rightarrow$$

$$\rightarrow\quad\boxed{I=3\ln|x|+\frac{2}{x}+5\ln|x-2|-7\ln|x+2|+k}$$

k) $\int \frac{x^3-4x^2+4x}{x^4-2x^3-4x^2+8x}dx$

Como *grado numerador* < *grado denominador*, factorizamos el denominador:

$$x^4-2x^3-4x^2+8x = x\cdot\left(x^3-2x^2-4x+8\right) = x\cdot(x-2)\cdot\left(x^2-4\right) = x\cdot(x-2)^2\cdot(x+2)$$

	1	-2	-4	8
2		2	0	-8
	1	0	-4	0

$$\frac{x^3-4x^2+4x}{x^4-2x^3-4x^2+8x} = \frac{A}{x}+\frac{B}{x-2}+\frac{C}{(x-2)^2}+\frac{D}{x+2} =$$

$$= \frac{A\cdot(x-2)^2\cdot(x+2)+Bx\cdot(x-2)\cdot(x+2)+Cx\cdot(x+2)+Dx\cdot(x-2)^2}{x\cdot(x-2)^2\cdot(x+2)} \rightarrow$$

$$\rightarrow \left\langle \begin{array}{lll} x=0 & \rightarrow\ 0=8A & \rightarrow \\ x=-2 & \rightarrow\ -32=-32D & \rightarrow \\ x=2 & \rightarrow\ 0=8C & \rightarrow \\ \left(x^3\right) & \rightarrow\ 1=A+B+D & \xrightarrow{1=0+B+1} \end{array} \right. \boxed{\begin{array}{l} A=0 \\ D=1 \\ C=0 \\ B=0 \end{array}}$$

$$\int \frac{x^3-4x^2+4x}{x^4-2x^3-4x^2+8x}dx = \int \frac{1}{x+2}dx \rightarrow \boxed{I=\ln|x+2|+k}$$

Pero también podíamos haber llegado a la conclusión más fácilmente si nos hubiesemos parado a factorizar el numerador previamente:

$$x^3-4x^2+4x = x\cdot\left(x^2-4x+4\right) = x\cdot(x+2)\cdot(x-2) \rightarrow$$

$$\rightarrow \frac{x^3-4x^2+4x}{x^4-2x^3-4x^2+8x} = \frac{\cancel{x}\cdot\cancel{(x-2)^2}}{\cancel{x}\cdot\cancel{(x-2)^2}\cdot(x+2)} = \frac{1}{x+2} \rightarrow I=\int\frac{1}{x+2}dx \rightarrow$$

$$\boxed{I=\ln|x+2|+k}$$

l) $I=\int \frac{x^4+2x-6}{x^3+x^2-2x}\,dx$

Como *grado numerador* > *grado denominador*, dividimos:

$$
\begin{array}{rrrrr|l}
\cancel{x^4} & & & +2x & -6 & x^3+x^2-2x \\
\cancel{x^4} & -x^3 & +2x^2 & & & \overline{x-1} \\
\hline
 & \cancel{-x^3} & +2x^2 & \cancel{+2x} & +1 & \\
 & \cancel{x^3} & +x^2 & \cancel{-2x} & & \\
\hline
 & & 3x^2 & & -6 &
\end{array}
$$

$$I=\int\left(x-1+\frac{3x^2-6}{x^3+x^2-2x}\right)dx \xrightarrow[\text{el denominador}]{\text{Factorizamos}} x^3+x^2-2x=x\cdot(x-1)\cdot(x+2)$$

$$\frac{3x^2-6}{x^3+x^2-2x}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+2}=\frac{A\cdot(x-1)\cdot(x+2)+Bx\cdot(x+2)+Cx\cdot(x-1)}{x\cdot(x-1)\cdot(x+2)} \rightarrow$$

$$\rightarrow \left\langle \begin{array}{lll} x=0 & \rightarrow & -6=-2A \rightarrow \\ x=1 & \rightarrow & -3=3B \rightarrow \\ x=-2 & \rightarrow & 6=6C \rightarrow \end{array}\right. \boxed{\begin{array}{l} A=3 \\ B=-1 \\ C=1 \end{array}}$$

$$I=\int\left(x-1+\frac{3}{x}-\frac{1}{x-1}+\frac{1}{x+2}\right)dx \rightarrow \boxed{I=\frac{x^2}{2}-x+3\cdot\ln|x|-\ln|x-1|+\ln|x+2|+k}$$

m) $I=\int \frac{5x^2}{x^3-3x^2+3x-1}\,dx$

Como *grado numerador* < *grado denominador*, factorizamos el denominador:

$$
\begin{array}{r|rrrr|}
 & 1 & -3 & 3 & -1 \ \ \boxed{0} \\
1 & & 1 & -2 & 1 \\
\hline
 & 1 & -2 & 1 & \boxed{0} \\
1 & & 1 & -1 & \\
\hline
 & 1 & -1 & \boxed{0} &
\end{array}
\rightarrow x^3-3x^2+3x-1=(x-1)^3
$$

$$\frac{5x^2}{x^3-3x^2+3x-1}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}=\frac{A\cdot(x-1)^2+B\cdot(x-1)+C}{(x-1)^3} \rightarrow$$

$$\rightarrow \left\langle \begin{array}{lllll} x=1 & \rightarrow & 5=C & \rightarrow \\ (x^2) & \rightarrow & 5=A & \rightarrow \\ T.I. & \rightarrow & 0=A-B+C & \rightarrow \end{array}\right. \boxed{\begin{array}{l} A=5 \\ B=10 \\ C=5 \end{array}}$$

$$I=\int \frac{5}{x-1}+\frac{10}{(x-1)^2}+\frac{5}{(x-1)^3}dx=5\int\frac{1}{x-1}dx+10\int(x-1)^{-2}dx+5\int(x-1)^{-3}dx \rightarrow$$

$$\rightarrow 5\cdot\ln|x-1|-10\cdot(x-1)^{-1}-\frac{5}{2}\cdot(x-1)^{-2}+k \rightarrow \boxed{I=5\cdot\ln|x-1|-\frac{10}{x-1}-\frac{5}{2(x-1)^2}+k}$$

n) $I=\int\frac{x^3}{x^2-1}dx$

Como *grado numerador* > *grado denominador*, dividimos:

$$\begin{array}{rr|l} \cancel{x^3} & & x^2-1 \\ \cline{3-3} \cancel{-x^3} & +x & x \\ \hline & x & \end{array}$$

$$\frac{x^3}{x^2-1}=x+\frac{x}{x^2-1} \rightarrow I=\int\frac{x^3}{x^2-1}dx=\int\left(x+\frac{x}{x^2-1}\right)dx$$

$$\frac{x^3}{x^2-1}=\frac{A}{x+1}+\frac{B}{x-1}=\frac{A\cdot(x-1)+B\cdot(x+1)}{(x+1)\cdot(x-1)} \quad \begin{cases} x=-1 \rightarrow -1=-2A \rightarrow \\ x=1 \rightarrow 1=2B \rightarrow \end{cases} \boxed{\begin{array}{l} A=1/2 \\ B=1/2 \end{array}}$$

$$I=\int\left(x+\frac{1/2}{x+1}+\frac{1/2}{x-1}\right)dx=\int\left(x+\frac{1/2}{x+1}+\frac{1/2}{x-1}\right)dx=\frac{x^2}{2}+\frac{1}{2}\cdot\ln(x+1)+\frac{1}{2}\cdot\ln(x-1)+k=$$

$$=\frac{1}{2}\cdot\left[x^2+\ln(x+1)+\ln(x-1)\right]+k=\frac{1}{2}\cdot\left[x^2+\ln(x+1)\cdot(x-1)\right]+k \rightarrow$$

$$\rightarrow \boxed{I=\frac{1}{2}\cdot\left[x^2+\ln\left(x^2-1\right)\right]+k}$$

o) $I=\int\frac{3x-2}{x^2-4}dx$

Como *grado numerador* < *grado denominador*, factorizamos el denominador:

$$\frac{3x-2}{x^2-4}=\frac{A}{x+2}+\frac{B}{x-2}=\frac{A\cdot(x-2)+B\cdot(x+2)}{(x+2)\cdot(x-2)} \quad \begin{cases} x=-2 \rightarrow -8=-4A \rightarrow \\ x=2 \rightarrow 4=4B \rightarrow \end{cases} \boxed{\begin{array}{l} A=2 \\ B=1 \end{array}}$$

$$I=\int\frac{3x-2}{x^2-4}dx=\int\left(\frac{2}{x+2}+\frac{1}{x-2}\right)dx \rightarrow \boxed{I=2\ln(x+2)+\ln(x-2)+k}$$

INTEGRALES POR PARTES

Int-7 a) $\int x \cdot e^{4x}\, dx$ b) $\int x^2 \cdot e^x\, dx$ c) $\int (x-1)\cdot e^x\, dx$d)

d) $\int e^x \cdot (x^2 - 2x - 1)\, dx$ e) $\int x^2 \cdot e^{-2x}\, dx$ f) $\int x^3 \cdot e^{-x^2}\, dx$

g) $\int x^5 \cdot e^{-x^3}\, dx$ h) $\int e^x \cdot \operatorname{sen} x\, dx$ i) $\int e^x \cdot \cos x\, dx$

j) $\int \ln x\, dx$ k) $\int (\ln x)^2\, dx$ l) $\int x^3 \ln x\, dx$

m) $\int x \cdot \operatorname{sen} x\, dx$ n) $\boxed{\int x \cdot \cos x\, dx}$ o) $\int x^2 \cdot \operatorname{sen} x\, dx$

p) $\int x^2 \cdot \cos x\, dx$ q) $\int x^2 \cdot \operatorname{sen} 2x\, dx$ r) $\int x^2 \cdot \cos 2x\, dx$

s) $\int \frac{x}{\cos^2 x}\, dx$ t) $\int \operatorname{arcsen} x\, dx$ u) $\int \arccos x\, dx$

v) $\int \operatorname{arctg} x\, dx$ w) $\int x \cdot \operatorname{arctg} x\, dx$ x) $\int \cos(\ln x)\, dx$

y) $\int \operatorname{sen}^2 x\, dx$ z) $\int \frac{\ln \sqrt{x}}{\sqrt{x}}\, dx$ α) $\int 2^x \cdot \operatorname{sen} x\, dx$

β) $\int e^{-2x} \cdot (2x+1)^2\, dx$ γ) $\int \sqrt{x} \cdot \ln x\, dx$

Solución:

a) $\int x \cdot e^{4x}\, dx = \left\{ \begin{array}{lll} u = x & \rightarrow & du = dx \\ dv = e^{4x}\, dx & \rightarrow & v = \frac{1}{4} e^{4x} \end{array} \right\} = \frac{1}{4} x e^{4x} - \int \frac{1}{4} e^{4x}\, dx =$

$$= \frac{1}{4} x e^{4x} - \frac{1}{16} e^{4x} + k = \frac{1}{4} e^{4x} \cdot \left(x - \frac{1}{4} \right) + k$$

$$\textbf{b)}\int x^2\cdot e^x\,dx=\left\{\begin{matrix} u=x^2 & \to & du=2x\,dx \\ dv=e^x\,dx & \to & v=e^x \end{matrix}\right\}=x^2\cdot e^x-\int 2x\cdot e^x\,dx=\left\{\begin{matrix} u=2x & \to & du=2\,dx \\ dv=e^x\,dx & \to & v=e^x \end{matrix}\right\}=$$

$$=x^2\cdot e^x-\left[2x\cdot e^x-\int 2e^x\,dx\right]=x^2\cdot e^x-2x\cdot e^x+2e^x+k=e^x\cdot\left(x^2-2x+2\right)+k$$

$$\textbf{c)}\int(x-1)\cdot e^x\,dx=\left\{\begin{matrix} u=x-1 & \to & du=dx \\ dv=e^x dx & \to & v=e^x \end{matrix}\right\}=(x-1)\cdot e^x-\int e^x\,dx=(x-1)\cdot e^x-e^x+k=$$

$$=e^x\cdot(x-1-1)+k=e^x\cdot(x-2)+k$$

$$\textbf{d)}\int e^x\cdot\left(x^2-2x-1\right)dx=\left\{\begin{matrix} u=x^2-2x-1 & \to & du=(2x-2)\,dx \\ dv=e^x dx & \to & v=e^x \end{matrix}\right\}=\left(x^2-2x-1\right)\cdot e^x-$$

$$-\int e^x\cdot(2x-2)\,dx=\left\{\begin{matrix} u=2x-2 \to & du=2\,dx \\ dv=e^x dx \to & v=e^x \end{matrix}\right\}=\left(x^2-2x-1\right)\cdot e^x-$$

$$-\left[(2x-2)\cdot e^x-\int 2e^x\,dx\right]=\left(x^2-2x-1\right)\cdot e^x-(2x-2)\cdot e^x+2e^x+k=$$

$$=e^x\cdot\left(x^2-2x-1-2x+2+2\right)+k=e^x\cdot\left(x^2-4x+3\right)+k$$

$$\textbf{e)}\int x^2\cdot e^{-2x}\,dx=\left\{\begin{matrix} u=x^2 & \to & du=2x\,dx \\ dv=e^{-2x}\,dx & \to & v=-\frac{1}{2}e^{-2x} \end{matrix}\right\}=-\frac{1}{2}x^2\cdot e^{-2x}+\int x\cdot e^{-2x}\,dx=$$

$$=\left\{\begin{matrix} u=x & \to & du=dx \\ dv=e^{-2x}\,dx & \to & v=-\frac{1}{2}e^{-2x} \end{matrix}\right\}=-\frac{1}{2}x^2\cdot e^{-2x}+\left[-\frac{1}{2}x\cdot e^{-2x}+\int\frac{1}{2}e^{-2x}\,dx\right]=$$

$$=-\frac{1}{2}x^2\cdot e^{-2x}-\frac{1}{2}x\cdot e^{-2x}-\frac{1}{4}e^{-2x}+k=e^{-2x}\cdot\left(-\frac{1}{2}x^2-\frac{1}{2}x-\frac{1}{4}\right)+k$$

$$\textbf{f)}\int x^3\cdot e^{-x^2}\,dx=\left\{\begin{matrix} u=x^2 & \to & du=2x\,dx \\ dv=x\cdot e^{-x^2}dx & \to & v=-\frac{1}{2}e^{-x^2} \end{matrix}\right\}=-\frac{1}{2}x^2\cdot e^{-x^2}+\int x\cdot e^{-x^2}\,dx=$$

$$=-\frac{1}{2}x^2\cdot e^{-x^2}-\frac{1}{2}e^{-x^2}+k=-\frac{1}{2}e^{-x^2}\cdot\left(x^2+1\right)+k$$

$$\textbf{g)}\int x^5\cdot e^{-x^3}\,dx=\left\{\begin{matrix} u=x^3 & \to & du=3x^2\,dx \\ dv=x^2\cdot e^{-x^3}dx \to & & v=-\frac{1}{3}e^{-x^3} \end{matrix}\right\}=-\frac{1}{3}x^3\cdot e^{-x^3}+\int x^2e^{-x^3}\,dx=$$

$$=-\frac{1}{3}x^3\cdot e^{-x^3}-\frac{1}{3}e^{-x^3}+k=-\frac{1}{3}e^{-x^3}\cdot\left(x^3+1\right)+k$$

h) $\int e^x \cdot \operatorname{sen} x\, dx = \left\{\begin{matrix} u = \operatorname{sen} x & \to & du = \cos x\, dx \\ dv = e^x\, dx & \to & v = e^x \end{matrix}\right\} = e^x \cdot \operatorname{sen} x - \int e^x \cdot \cos x =$

$= \left\{\begin{matrix} u = \cos x & \to & du = -\operatorname{sen} x\, dx \\ dv = e^x\, dx & \to & v = e^x \end{matrix}\right\} = e^x \cdot \operatorname{sen} x - \left[e^x \cdot \cos x + \int e^x \cdot \operatorname{sen} x\, dx \right] =$

$= e^x \cdot (\operatorname{sen} x - \cos x) - \int e^x \cdot \operatorname{sen} x\, dx \quad \to \quad 2\int e^x \cdot \operatorname{sen} x\, dx = e^x \cdot (\operatorname{sen} x - \cos x) + k \quad \to$

$\to \quad \int e^x \cdot \operatorname{sen} x\, dx = \frac{1}{2} e^x \cdot (\operatorname{sen} x - \cos x) + k$

i) $\int e^x \cdot \cos x\, dx = \left\{\begin{matrix} u = \cos x & \to & du = -\operatorname{sen} x\, dx \\ dv = e^x\, dx & \to & v = e^x \end{matrix}\right\} = e^x \cdot \cos x + \int e^x \cdot \operatorname{sen} x\, dx =$

$= \left\{\begin{matrix} u = \operatorname{sen} x & \to & du = \cos x\, dx \\ dv = e^x\, dx & \to & v = e^x \end{matrix}\right\} = e^x \cdot \cos x + e^x \cdot \operatorname{sen} x - \int e^x \cdot \cos x\, dx =$

$= e^x \cdot (\cos x + \operatorname{sen} x) - \int e^x \cdot \cos x\, dx \quad \to \quad 2\int e^x \cdot \cos x\, dx = e^x \cdot (\cos x + \operatorname{sen} x) + k \quad \to$

$\to \quad \int e^x \cdot \cos x\, dx = \frac{1}{2} e^x \cdot (\cos x + \operatorname{sen} x) + k$

j) $\int \ln x\, dx = \left\{\begin{matrix} u = \ln x & \to & du = \frac{1}{x} dx \\ dv = dx & \to & v = x \end{matrix}\right\} = x \cdot \ln x - \int dx = x \cdot \ln x - x + k = x \cdot (\ln x - 1) + k$

k) $\int (\ln x)^2\, dx = \left\{\begin{matrix} u = \ln x & \to & du = \frac{1}{x} dx \\ dv = \ln x\, dx & \xrightarrow[\int \ln x\, dx = x \cdot (\ln x - 1)]{\text{Del apdo. k)}} & v = x \cdot (\ln x - 1) \end{matrix}\right\} = x \cdot (\ln x - 1) \cdot \ln x -$

$- \int \cancel{x} \cdot (\ln x - 1) \cdot \frac{1}{\cancel{x}} dx = x \cdot (\ln x - 1) \cdot \ln x - \int (\ln x - 1)\, dx = x \cdot (\ln x - 1) \cdot \ln x -$

$- \left[x \cdot (\ln x - 1) - x \right] + k = x \cdot (\ln x - 1)^2 + x + k$

l) $\int x^3 \cdot \ln x\, dx = \left\{\begin{matrix} u = \ln x & \to & du = \frac{1}{x} dx \\ dv = x^3\, dx & \to & v = \frac{1}{4} x^4 \end{matrix}\right\} = \frac{1}{4} x^4 \cdot \ln x - \int \frac{1}{4} x^{\cancel{4}\,3} \cdot \frac{1}{\cancel{x}} dx =$

$= \frac{1}{4} x^4 \cdot \ln x - \frac{1}{4} \int x^3\, dx = = \frac{1}{4} x^4 \cdot \ln x - \frac{1}{4} \frac{x^3}{4} + k = \frac{1}{4} x^3 \cdot \left(x \ln x - \frac{1}{4} \right) + k$

m) $\int x \cdot \operatorname{sen} x\, dx = \left\{\begin{matrix} u = x & \to & du = dx \\ dv = \operatorname{sen} x\, dx & \to & v = -\cos x \end{matrix}\right\} = -x \cdot \cos x + \int \cos x\, dx = -x \cdot \cos x + \operatorname{sen} x + k$

$$\boxed{\textbf{n)}\int x\cdot\cos x\,dx=\left\{\begin{matrix}u=x & \to & du=dx\\ dv=\cos x\,dx & \to & v=\operatorname{sen}x\end{matrix}\right\}=x\cdot\operatorname{sen}x-\int\operatorname{sen}x\,dx=x\cdot\operatorname{sen}x+\cos x+k}$$

$$\textbf{o)}\int x^2\cdot\operatorname{sen}x\,dx=\left\{\begin{matrix}u=x^2 & \to & du=2x\,dx\\ dv=\operatorname{sen}x\,dx & \to & v=-\cos x\end{matrix}\right\}=-x^2\cos x+\int 2x\cos x\,dx=$$

$$=\left\{\begin{matrix}u=2x & \to & du=2dx\\ dv=\cos x & \to & v=\operatorname{sen}x\end{matrix}\right\}=-x^2\cos x+\left[2x\operatorname{sen}x-\int 2\operatorname{sen}x\,dx\right]=$$

$$=-x^2\cos x+2x\operatorname{sen}x+2\cos x+k$$

$$\textbf{p)}\int x^2\cdot\cos x\,dx=\left\{\begin{matrix}u=x^2 & \to & du=2x\,dx\\ dv=\cos x\,dx & \to & v=\operatorname{sen}x\end{matrix}\right\}=x^2\cdot\operatorname{sen}x-\int 2x\cdot\operatorname{sen}x\,dx=$$

$$=\left\{\begin{matrix}u=2x & \to & du=2\,dx\\ dv=\operatorname{sen}x\,dx & \to & v=-\cos x\end{matrix}\right\}=x^2\cdot\operatorname{sen}x-\left[-2x\cdot\cos x+\int 2\cos x\,dx\right]=$$

$$x^2\cdot\operatorname{sen}x+2x\cdot\cos x-2\operatorname{sen}x+k$$

$$\textbf{q)}\int x^2\cdot\operatorname{sen}2x\,dx=\left\{\begin{matrix}u=x^2 & \to & du=2x\,dx\\ dv=\operatorname{sen}2x\,dx & \to & v=-\frac{1}{2}\cos 2x\end{matrix}\right\}=-\frac{1}{2}x^2\cdot\cos 2x+\int x\cdot\cos 2x\,dx=$$

$$=\left\{\begin{matrix}u=x & \to & du=dx\\ dv=\cos 2x & \to & v=\frac{1}{2}\operatorname{sen}2x\end{matrix}\right\}=-\frac{1}{2}x^2\cdot\cos 2x+$$

$$+\left[\frac{1}{2}x\cdot\operatorname{sen}2x-\int\frac{1}{2}\operatorname{sen}2x\,dx\right]=-\frac{1}{2}x^2\cdot\cos 2x+\frac{1}{2}x\cdot\operatorname{sen}2x+\frac{1}{4}\cos 2x+k$$

$$\textbf{r)}\int x^2\cdot\cos 2x\,dx=\left\{\begin{matrix}u=x^2 & \to & du=2x\,dx\\ dv=\cos 2x\,dx & \to & v=\frac{1}{2}\operatorname{sen}2x\end{matrix}\right\}=\frac{1}{2}x^2\cdot\operatorname{sen}2x-\int x\cdot\operatorname{sen}2x\,dx=$$

$$=\left\{\begin{matrix}u=x & \to & du=dx\\ dv=\operatorname{sen}2x & \to & v=-\frac{1}{2}\cos 2x\end{matrix}\right\}=\frac{1}{2}x^2\cdot\operatorname{sen}2x-$$

$$-\left[-\frac{1}{2}x\cdot\cos 2x+\int\frac{1}{2}\cos 2x\,dx\right]=\frac{1}{2}x^2\cdot\operatorname{sen}2x+\frac{1}{2}x\cdot\cos 2x-\frac{1}{4}\operatorname{sen}2x+k$$

$$\textbf{s)}\int\frac{x}{\cos^2 x}\,dx=\int x\cdot\sec^2 x\,dx=\left\{\begin{matrix}u=x & \to & du=dx\\ dv=\sec^2 x\,dx & \to & v=\operatorname{tg}x\end{matrix}\right\}=x\cdot\operatorname{tg}x-\int\operatorname{tg}x\,dx=$$

$$=x\cdot\operatorname{tg}x-\int\frac{\operatorname{sen}x}{\cos x}\,dx=x\cdot\operatorname{tg}x+\ln\left(\cos x\right)+k$$

t) $\int \arcsin x\,dx = \left\{\begin{array}{lll} u = \operatorname{arcsen} x & \rightarrow & du = \dfrac{dx}{\sqrt{1-x^2}} \\ dv = dx & \rightarrow & v = x \end{array}\right\} = x\cdot\operatorname{arcsen} x - \int \dfrac{x}{\sqrt{1-x^2}}\,dx =$

$$= x\cdot\operatorname{arcsen} x + \frac{1}{2}\int -2x\cdot\left(1-x^2\right)^{-1/2} dx = x\cdot\operatorname{arcsen} x + \frac{\cancel{1}}{\cancel{2}}\cdot\frac{\left(1-x^2\right)^{1/2}}{\cancel{1/2}} + k =$$

$$= x\cdot\operatorname{arcsen} x + \left(1-x^2\right)^{1/2} + k == x\cdot\operatorname{arcsen} x + \sqrt{1-x^2} + k$$

u) $\int \arccos x\,dx = \left\{\begin{array}{lll} u = \arccos x & \rightarrow & du = -\dfrac{dx}{\sqrt{1-x^2}} \\ dv = dx & \rightarrow & v = x \end{array}\right\} = x\cdot\arccos x + \int \dfrac{x}{\sqrt{1-x^2}}\,dx =$

$$= x\cdot\arccos x - \frac{1}{2}\int -2x\cdot\left(1-x^2\right)^{-1/2} dx = x\cdot\arccos x - \frac{\cancel{1}}{\cancel{2}}\cdot\frac{\left(1-x^2\right)^{1/2}}{\cancel{1/2}} + k =$$

$$= x\cdot\arccos x - \left(1-x^2\right)^{1/2} + k == x\cdot\arccos x - \sqrt{1-x^2} + k$$

v) $\int \operatorname{arctg} x\,dx = \left\{\begin{array}{lll} u = \operatorname{arctg} x & \rightarrow & du = \dfrac{dx}{1+x^2} \\ dv = dx & \rightarrow & v = x \end{array}\right\} = x\cdot\operatorname{arctg} x - \int \dfrac{x}{1+x^2}\,dx =$

$$= x\cdot\operatorname{arctg} x - \frac{1}{2}\int \frac{2x}{1+x^2}\,dx = x\cdot\operatorname{arctg} x - \frac{1}{2}\ln\left(1+x^2\right) + k$$

w) $\int x\cdot\operatorname{arctg} x\,dx = \left\{\begin{array}{lll} u = \operatorname{arctg} x & \rightarrow & du = \dfrac{dx}{1+x^2} \\ dv = x\,dx & \rightarrow & v = \dfrac{x^2}{2} \end{array}\right\} = \dfrac{x^2}{2}\cdot\operatorname{arctg} x - \dfrac{1}{2}\int \dfrac{x^2}{1+x^2}\,dx =$

$$= \left\{\frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}\right\} = \frac{x^2}{2}\cdot\operatorname{arctg} x - \frac{1}{2}\cdot\left[\int dx - \int \frac{dx}{1+x^2}\right] =$$

$$= \frac{x^2}{2}\cdot\operatorname{arctg} x - \frac{x}{2} + \frac{1}{2}\operatorname{arctg} x + k = \frac{1+x^2}{2}\cdot\operatorname{arctg} x + \frac{x}{2} + k$$

x) $\displaystyle\int \cos(\ln x)\,dx = \left\{\begin{matrix} u=\cos(\ln x) & \rightarrow & du=-\frac{1}{x}\operatorname{sen}(\ln x)\,dx \\ dv=dx & \rightarrow & v=x \end{matrix}\right\} = x\cdot\cos(\ln x)+$

$$+\int \frac{1}{\cancel{x}}\cdot\cancel{x}\cdot\operatorname{sen}(\ln x)\,dx = \left\{\begin{matrix} u=\operatorname{sen}(\ln x) & \rightarrow & du=\frac{1}{x}\cos(\ln x)\,dx \\ dv=dx & \rightarrow & v=x \end{matrix}\right\} =$$

$$= x\cdot\cos(\ln x)+x\cdot\operatorname{sen}(\ln x)-\int\frac{1}{\cancel{x}}\cdot\cancel{x}\cdot\cos(\ln x)\,dx+k \rightarrow$$

$$\rightarrow I = x\cdot\left[\cos(\ln x)+\operatorname{sen}(\ln x)\right]+k-I \rightarrow 2I = x\cdot\left[\cos(\ln x)+\operatorname{sen}(\ln x)\right]+k \rightarrow$$

$$\rightarrow \quad I=\frac{x}{2}\cdot\left[\cos(\ln x)+\operatorname{sen}(\ln x)\right]+k$$

y) $\displaystyle\int \operatorname{sen}^2 x\,dx = \left\{\begin{matrix} u=\operatorname{sen}^2 x & \rightarrow & du=2\operatorname{sen}x\cdot\cos x\,dx \\ dv=dx & \rightarrow & v=x \end{matrix}\right\} = x\cdot\operatorname{sen}^2 x-\int 2x\operatorname{sen}x\cdot\cos x\,dx=$

$$\overbrace{====}^{\operatorname{sen}2x=2\operatorname{sen}x\cdot\cos x}\quad x\cdot\operatorname{sen}^2 x-\int x\cdot\operatorname{sen}2x\,dx = \left\{\begin{matrix} u=x & \rightarrow & du=dx \\ dv=\operatorname{sen}2x\,dx & \rightarrow & v=-\frac{1}{2}\cos 2x \end{matrix}\right\} =$$

$$= x\cdot\operatorname{sen}^2 x-\left(-\frac{x}{2}\cos 2x+\frac{1}{2}\int\cos 2x\,dx\right) = x\cdot\operatorname{sen}^2 x+\frac{x}{2}\cos 2x-\frac{1}{4}\operatorname{sen}2x+k=$$

$$= x\cdot\left[\operatorname{sen}^2 x+\frac{1}{2}\left(\cos^2 x-\operatorname{sen}^2 x\right)\right]-\frac{1}{4}\operatorname{sen}2x+k = \frac{x}{2}\cdot\underbrace{\left(\operatorname{sen}^2 x+\cos^2 x\right)}_{\operatorname{sen}^2 x+\cos^2 x=1}-\frac{1}{4}\operatorname{sen}2x+k$$

$$\frac{x}{2}-\frac{1}{4}\operatorname{sen}2x+k$$

y) $\displaystyle\int \operatorname{sen}^2 x\,dx = \left\{\begin{matrix} u=\operatorname{sen}x & \rightarrow & du=\cos x\,dx \\ dv=\operatorname{sen}x\,dx & \rightarrow & v=-\cos x \end{matrix}\right\} = -\operatorname{sen}x\cdot\cos x+\int\cos^2 x\,dx=$

$$\overbrace{====}^{\cos^2 x=1-\operatorname{sen}^2 x}\quad -\operatorname{sen}x\cdot\cos x+\int\left(1-\operatorname{sen}^2 x\right)dx = -\operatorname{sen}x\cdot\cos x+x-\int\operatorname{sen}^2 x\,dx \rightarrow$$

$$\rightarrow \quad I=-\operatorname{sen}x\cdot\cos x+x-I \quad\rightarrow\quad 2I=-\operatorname{sen}x\cdot\cos x+x \quad\rightarrow$$

$$\rightarrow \quad I=-\frac{1}{2}\operatorname{sen}x\cdot\cos x+\frac{x}{2}+k \quad \overbrace{====}^{\operatorname{sen}2x=2\operatorname{sen}x\cdot\cos x}\quad \frac{x}{2}-\frac{1}{4}\operatorname{sen}2x+k$$

Hemos retocado el resultado final de ambas ecuaciones para demostrar que estamos ante la misma solución haciéndolo de una u otra manera. Esta integral también está resuelta en el apartado de “Integrales trigonométricas” por un tercer método.

z) $\displaystyle\int \frac{\ln\sqrt{x}}{\sqrt{x}}\,dx = \left\{\begin{array}{lll} u = \ln\sqrt{x} & \rightarrow & du = \dfrac{\frac{1}{2\sqrt{x}}}{\sqrt{x}}\,dx = \dfrac{1}{2x}\,dx \\ dv = \sqrt{x}\,dx & \rightarrow & v = \dfrac{2}{3}\sqrt{x^3} = \dfrac{2}{3}x\sqrt{x} \end{array}\right\} = \frac{2}{3}\sqrt{x^3}\cdot\ln\sqrt{x} -$

$$-\int \frac{2}{3}\cancel{x}\sqrt{x}\cdot\frac{1}{2\cancel{x}}\,dx = \frac{2}{3}\sqrt{x^3}\cdot\ln\sqrt{x} - \frac{1}{3}\int\sqrt{x}\,dx = \frac{2}{3}\sqrt{x^3}\cdot\ln\sqrt{x} - \frac{2}{9}\sqrt{x^3} + k =$$

$$= \frac{2}{3}\sqrt{x^3}\cdot\left(\ln\sqrt{x} - \frac{1}{3}\right) + k$$

α) $\displaystyle\int 2^x\cdot\operatorname{sen}x\,dx = \left\{\begin{array}{lll} u = 2^x & \rightarrow & du = 2^x\cdot\ln 2\,dx \\ dv = \operatorname{sen}x\,dx & \rightarrow & v = -\cos x \end{array}\right\} = -2^x\cdot\cos x + \int 2^x\cdot\ln 2\cdot\cos x =$

$$= \left\{\begin{array}{lll} u = 2^x & \rightarrow & du = 2^x\cdot\ln 2\,dx \\ dv = \cos x\,dx & \rightarrow & v = \operatorname{sen}x \end{array}\right\} = -2^x\cdot\cos x + \ln 2\cdot\left[2^x\cdot\operatorname{sen}x - \int 2^x\cdot\ln 2\cdot\operatorname{sen}x\right] =$$

$$= 2^x\cdot(\ln 2\cdot\operatorname{sen}x - \cos x) - (\ln 2)^2\cdot\int 2^x\cdot\operatorname{sen}x\,dx \quad\rightarrow$$

$$\rightarrow \quad \left[1 + (\ln 2)^2\right]\cdot\int 2^x\cdot\operatorname{sen}x\,dx = 2^x\cdot(\ln 2\cdot\operatorname{sen}x - \cos x) \quad\rightarrow$$

$$\rightarrow \quad \int 2^x\cdot\operatorname{sen}x\,dx = \frac{2^x\cdot(\ln 2\cdot\operatorname{sen}x - \cos x)}{1 + (\ln 2)^2} + k$$

β) $\displaystyle\int e^{-2x}\cdot(2x+1)^2\,dx = \left\{\begin{array}{lll} u = (2x+1)^2 & \rightarrow & du = 4\cdot(2x+1) \\ dv = e^{-2x}\,dx & \rightarrow & v = -\dfrac{1}{2}e^{-2x} \end{array}\right\} = -\frac{1}{2}e^{-2x}\cdot(2x+1)^2 +$

$$+\int \frac{1}{2}e^{-2x}\cdot 4\cdot(2x+1)\,dx = \left\{\begin{array}{lll} u = (2x+1) & \rightarrow & du = 2 \\ dv = 2e^{-2x}\,dx & \rightarrow & v = -e^{-2x} \end{array}\right\} =$$

$$= -\frac{1}{2}e^{-2x}\cdot(2x+1)^2 - 2e^{-2x}\cdot(2x+1) + \int 2e^{-2x}\,dx =$$

$$= -\frac{1}{2}e^{-2x}\cdot(2x+1)^2 - 2e^{-2x}\cdot(2x+1) - e^{-2x} + k = e^{-2x}\cdot\left(-2x^2 - 2x - \frac{1}{2} - 2x - 1 - 1\right) + k =$$

$$= e^{-2x}\cdot\left(-2x^2 - 4x - \frac{5}{2}\right) + k$$

$$\gamma)\int \sqrt{x}\cdot \ln x\,dx = \left\{\begin{array}{lll} u=\ln x & \rightarrow & du=\dfrac{1}{x} \\ dv=\sqrt{x}\,dx & \rightarrow & v=\dfrac{2\sqrt{x^3}}{3}=\dfrac{2x\sqrt{x}}{3}\end{array}\right\} = \frac{2x\sqrt{x}}{3}\cdot \ln x - \int \frac{2\cancel{x}\sqrt{x}}{3}\cdot\frac{1}{\cancel{x}}\,dx =$$

$$= \frac{2x\sqrt{x}}{3}\cdot \ln x - \int \frac{2\sqrt{x}}{3}\,dx = \frac{2x\sqrt{x}}{3}\cdot \ln x - \frac{2}{3}\cdot\frac{2x\sqrt{x}}{3} + k = \frac{2x\sqrt{x}}{3}\cdot\left(\ln x - \frac{2}{3}\right) + k$$

Zzzz falta por mirar del 1 al 24 de SM de segundo de bachillerato

Falta por meter las del libro rojo de SM de 2º de Bachillerato

Zzz seguir metiendo en el nº 23 e) de la página 351 del libro de Anaya