**Ejercicio 52**

*Calcula la razón y el término general de las siguientes progresiones geométricas:*

*a)* $a_5 = 256,\ r = 4$

*b)* $4,\ 12,\ 36,\ 108, \ldots$

*c)* $a_3 = 5,\ a_5 = 125$

*d)* $3,\ 3\sqrt{3},\ 9,\ 9\sqrt{3}, \ldots$

*e)* $-\frac{1}{2},\ -\frac{3}{2},\ -\frac{9}{2}, \ldots$

*f)* $-3,\ 12,\ -48,\ 192, \ldots$

**Solución.**

Para resolver el ejercicio utilizaremos las fórmulas $a_n = a_1 \cdot r^{n-1}$ y $a_q = a_p \cdot r^{q-p}$

a) $a_5 = 256,\ r = 4$

$$a_5 = a_1 \cdot r^{5-1} \implies 256 = a_1 \cdot 4^4 \implies a_1 = 1$$

$$a_n = a_1 \cdot r^{n-1} = 1 \cdot 4^{n-1} \implies \boxed{a_n = 4^{n-1}}$$

podemos hacerlo también de otra forma sin necesidad de calcular $a_1$

$$a_n = a_p \cdot r^{n-p} = a_5 \cdot r^{n-5} = 256 \cdot 4^{n-5} = \cancel{4^4} \cdot 4^{n-1} \cdot \cancel{4^{-4}} \implies a_n = 4^{n-1}$$

b) $4,\ 12,\ 36,\ 108, \ldots$

$$\begin{cases} a_1 = 4 \\ r = {}^{12}/_4 = 3 \end{cases} \implies a_n = a_1 \cdot r^{n-1} = 4 \cdot 3^{n-1} = 4 \cdot 3^n \cdot 3^{-1} \implies \boxed{a_n = \frac{4}{3} \cdot 3^n}$$

c) $a_3 = 5,\ a_5 = 125$

$$a_q = a_p \cdot r^{q-p} \implies a_5 = a_3 \cdot r^{5-3} \implies 125 = 5 \cdot r^2 \implies 25 = r^2 \implies r = \pm 5$$

$$a_n = a_p \cdot r^{n-p} = a_3 \cdot r^{n-3} \Rightarrow \begin{cases} a_n = 5 \cdot 5^{n-3} \implies \boxed{a_n = 5^{n-2}} \\ a_n = 5 \cdot (-5)^{n-3} \implies \boxed{a_n = -(-5)^{n-2}} \end{cases}$$

d) $3,\ 3\sqrt{3},\ 9,\ 9\sqrt{3}, \ldots$

$$\begin{cases} a_1 = 3 \\ r = {}^{3\sqrt{3}}/_3 = \sqrt{3} \end{cases} \Rightarrow a_n = a_1 \cdot r^{n-1} = 3 \cdot \left(\sqrt{3}\right)^{n-1} = 3 \cdot 3^{\frac{n-1}{2}} = 3^{\frac{n-1}{2}+1} \Rightarrow \boxed{a_n = 3^{\frac{n+1}{2}}}$$

e) $-\frac{1}{2},\ -\frac{3}{2},\ -\frac{9}{2}, \ldots$

$$\begin{cases} a_1 = -{}^1/_2 \\ r = \dfrac{-{}^3/_2}{-{}^1/_2} = 3 \end{cases} \implies a_n = a_1 \cdot r^{n-1} = \left(-\frac{1}{2}\right) \cdot 3^{n-1} = \left(-\frac{1}{2}\right) \cdot 3^n \cdot 3^{-1} \implies$$

$$\boxed{a_n = \left(-\frac{1}{6}\right) \cdot 3^n}$$

f) $-3,\ 12,\ -48,\ 192, \ldots$

$$\begin{cases} a_1 = -3 \\ r = {}^{12}/_{-3} = -4 \end{cases} \implies a_n = a_1 \cdot r^{n-1} = (-3) \cdot (-4)^{n-1} = (-3) \cdot (-4)^n \cdot (-4)^{-1}$$

$$\implies \boxed{a_n = \frac{3}{4} \cdot (-4)^n}$$