CÁLCULO INTUITIVO

LyC-1 Si $u(x) \to -1$ y $v(x) \to 0$ cuando $x \to +\infty$, calcula el $\lim\limits_{x\to+\infty}$ de:

a) $v(x) - u(x)$ b) $\dfrac{v(x)}{u(x)}$ c) $\log_2 \text{v(x)}$

d) $u(x) + v(x)$ e) $u(x) \cdot v(x)$ f) $\sqrt[3]{u(x)}$

Si $u(x) \to 2$ y $v(x) \to -3$ cuando $x \to +\infty$, calcula el $\lim\limits_{x\to+\infty}$ de:

g) $u(x) + v(x)$ h) $\dfrac{v(x)}{u(x)}$ i) $5^{\text{u(x)}}$

j) $\sqrt{u(x)}$ k) $u(x) \cdot v(x)$ l) $\sqrt[3]{u(x)}$

Solución:

a) $\lim\limits_{x\to+\infty} \left[v(x) - u(x)\right] = 0 - (-1) = 1$

b) $\lim\limits_{x\to+\infty} \left(\dfrac{v(x)}{u(x)}\right) = \dfrac{0}{-1} = 0$

c) $\lim\limits_{x\to+\infty} \left[\log_2 \text{v(x)}\right] =$ No existe

d) $\lim\limits_{x\to+\infty} \left[u(x) + v(x)\right] = -1 + 0 = -1$

e) $\lim\limits_{x\to+\infty} \left[u(x) \cdot v(x)\right] = -1 \cdot 0 = 0$

f) $\lim\limits_{x\to+\infty} \sqrt[3]{u(x)} = \sqrt[3]{-1} = -1$

g) $\lim\limits_{x\to+\infty} \left[u(x) + v(x)\right] = 2 - 3 = -1$

h) $\lim\limits_{x\to+\infty} \dfrac{v(x)}{u(x)} = \dfrac{-3}{2}$

i) $\lim\limits_{x\to+\infty} \left(5^{\text{u(x)}}\right) = 5^2 = 25$

j) $\lim\limits_{x\to+\infty} \sqrt{u(x)} = \sqrt{2}$

k) $\lim\limits_{x\to+\infty} \left[u(x) \cdot v(x)\right] = 2 \cdot (-3) = -6$

l) $\lim\limits_{x\to+\infty} \sqrt[3]{u(x)} = \sqrt[3]{2}$

LyC-2 Sin calcular halla los siguientes límites:

a) $\lim_{x\to+\infty}\left(x^2+3x-x^3\right)$ b) $\lim_{x\to+\infty}\left(-5\cdot 2^{2x}\right)$ c) $\lim_{x\to+\infty}\left(x^3-7x^2+3\right)$

d) $\lim_{x\to+\infty}\left(-5x^2+6x-2\right)$ e) $\lim_{x\to+\infty}\sqrt[3]{x^2+2}$ f) $\lim_{x\to+\infty}\left(-2\log x\right)$

g) $\lim_{x\to+\infty}\sqrt{5x^2-6x}$ h) $\lim_{x\to+\infty}(2.5)^{x+1}$ i) $\lim_{x\to+\infty}\left(-5\log_3 x\right)$

j) $\lim_{x\to+\infty}\left(4\log x\right)$ k) $3x^5-\sqrt{x}+1$ l) 0.5^x

m) -1.5^x n) $\log_2 x$ o) $\dfrac{1}{x^3+1}$

p) $\sqrt{x}$ q) 4^x r) 4^{-x}

s) $(-4)^x$ t) $\lim_{x\to+\infty}\left(x^2-\sqrt[3]{2x+1}\right)$ u) $\lim_{x\to+\infty}\left(x^2-2^x\right)$

v) $\lim_{x\to+\infty}\left(\sqrt{x^2+1}-\sqrt{x}\right)$ w) $\lim_{x\to+\infty}\left(3^x-2^x\right)$ x) $\lim_{x\to+\infty}\left(5^x-\sqrt[3]{x^8-2}\right)$

y) $\lim_{x\to+\infty}\left(\sqrt{x}-\log_5 x^4\right)$

Solución:

a) $\lim_{x\to+\infty}\left(x^2+3x-x^3\right)=-\infty$

b) $\lim_{x\to+\infty}\left(-5\cdot 2^{2x}\right)=-\infty$

c) $\lim_{x\to+\infty}\left(x^3-7x^2+3\right)=+\infty$

d) $\lim_{x\to+\infty}\left(-5x^2+6x-2\right)=-\infty$

e) $\lim_{x\to+\infty}\sqrt[3]{x^2+2}=+\infty$

f) $\lim_{x\to+\infty}\left(-2\log x\right)=-\infty$

g) $\lim_{x\to+\infty}\sqrt{5x^2-6x}=+\infty$

h) $\lim_{x\to+\infty}(2.5)^{x+1}=+\infty$

i) $\lim_{x\to+\infty}\left(-5\log_3 x\right)=-\infty$

j) $\lim_{x\to+\infty}\left(4\log x\right)=+\infty$

k) $\lim_{x\to+\infty}\left(3x^5-\sqrt{x}+1\right)=+\infty$

l) $\lim_{x\to+\infty}\left(0.5^x\right)=+\infty$

m) $\lim_{x\to+\infty}\left(-1.5^x\right)=\textit{No existe}$

n) $\lim_{x\to+\infty}\left(\log_2 x\right)=+\infty$

o) $\lim_{x\to+\infty}\left(\dfrac{1}{x^3+1}\right)=0$

p) $\lim_{x\to+\infty}\sqrt{x}=+\infty$

q) $\lim_{x\to+\infty}\left(4^x\right)=+\infty$

r) $\lim_{x\to+\infty}\left(4^{-x}\right)=0$

s) $\lim\limits_{x\to+\infty} (-4)^x = \textit{No existe}$

t) $\lim\limits_{x\to+\infty} \left(x^2 - \sqrt[3]{2x+1}\right) = +\infty$

u) $\lim\limits_{x\to+\infty} \left(x^2 - 2^x\right) = -\infty$

v) $\lim\limits_{x\to+\infty} \left(\sqrt{x^2+1} - \sqrt{x}\right) = +\infty$

w) $\lim\limits_{x\to+\infty} \left(3^x - 2^x\right) = +\infty$

x) $\lim\limits_{x\to+\infty} \left(5^x - \sqrt[3]{x^8-2}\right) = +\infty$

y) $\lim\limits_{x\to+\infty} \left(\sqrt{x} - \log_5 x^4\right) = +\infty$

a) Ordena de menor a mayor los siguientes infinitos:

$$\sqrt{x} \qquad 1.5^x \qquad x^2 \qquad 3x^5 \qquad 4^x \qquad \log_2 x$$

b) Teniendo en cuenta el resultado anterior calcula:

$$\lim_{x\to+\infty}\left(\frac{\log_2 x}{\sqrt{x}}\right) \qquad \lim_{x\to+\infty}\left(\frac{3x^5}{x^2}\right) \qquad \lim_{x\to+\infty}\left(\frac{\sqrt{x}}{1.5^x}\right)$$

Solución:

a) Orden de los infinitos:

$$\log_2 x \; < \; \sqrt{x} \; < \; x^2 \; < \; 3x^5 \; < \; 1.5^x \; < \; 4^x$$

N Resolver los siguientes límites:

$$\lim_{x\to+\infty}\left(\frac{\log_2 x}{\sqrt{x}}\right)=0 \qquad \lim_{x\to+\infty}\left(\frac{3x^5}{x^2}\right)=+\infty \qquad \lim_{x\to+\infty}\left(\frac{\sqrt{x}}{1.5^x}\right)=0$$

LyC-3 Cuando $x \to +\infty$ $f(x) \to +\infty$, $g(x) \to 4$ $h(x) \to -\infty$, $u(x) \to 0$, asigna, siempre que puedas, límite cuando $x \to +\infty$ a las expresiones siguientes:

a) $f(x)-h(x)$	b) $f(x)+h(x)$	c) $f(x)\cdot h(x)$	d) $f(x)^{f(x)}$
e) $f(x)^x$	f) $u(x)^{u(x)}$	g) $f(x)/h(x)$	h) $g(x)^{h(x)}$
i) $f(x)/u(x)$	j) $g(x)/u(x)$	k) $f(x)^{h(x)}$	l) $\left[-h(x)^{h(x)}\right]$
m) $u(x)/h(x)$	n) $x+f(x)$	o) $x+h(x)$	p) x^{-x}

Solución:

a) $\lim\limits_{x\to+\infty} \left[f(x)-h(x)\right] = (+\infty)-(-\infty) = (+\infty)$

b) $\lim\limits_{x\to+\infty} \left[f(x)+h(x)\right] = (+\infty)+(-\infty) = No\ existe$

c) $\lim\limits_{x\to+\infty} f(x)\cdot h(x) = (+\infty)\cdot(-\infty) = (-\infty)$

d) $\lim\limits_{x\to+\infty} f(x)^{f(x)} = (+\infty)^{(+\infty)} = (+\infty)$

e) $\lim\limits_{x\to+\infty} f(x)^{x} = (+\infty)^{(+\infty)} = (+\infty)$

f) $\lim\limits_{x\to+\infty} u(x)^{u(x)} = (0)^{(0)} = No\ existe$

g) $\lim\limits_{x\to+\infty} \dfrac{f(x)}{h(x)} = Depende\ del\ orden\ de\ los\ \inf initos$

h) $\lim\limits_{x\to+\infty} g(x)^{h(x)} = 4^{(-\infty)} = (0)$

i) $\lim\limits_{x\to+\infty} \dfrac{f(x)}{u(x)} = \dfrac{(+\infty)}{(0)} = (+\infty)$

j) $\lim\limits_{x\to+\infty} \dfrac{g(x)}{u(x)} = \dfrac{4}{(0)} = (+\infty)$

k) $\lim\limits_{x\to+\infty} f(x)^{h(x)} = (+\infty)^{(-\infty)} = \dfrac{1}{(+\infty)^{(+\infty)}} = \dfrac{1}{(+\infty)} = (0)$

l) $\lim\limits_{x\to+\infty} \left[-h(x)^{h(x)}\right] = (-\infty)^{(-\infty)} = No\ existe$

m) $\lim\limits_{x\to+\infty} \dfrac{u(x)}{h(x)} = \dfrac{(0)}{(-\infty)} = (0)$

n) **n)** $\lim\limits_{x\to+\infty} \left[x + f(x)\right] = (+\infty) + (+\infty) = (+\infty)$

o) $\lim\limits_{x\to+\infty} \left[x + h(x)\right] = (+\infty) - (+\infty) = No\ existe$

p) $\lim\limits_{x\to+\infty} \left(x^{-x}\right) = (+\infty)^{(-\infty)} = \dfrac{1}{(+\infty)^{(+\infty)}} = \dfrac{1}{(+\infty)} = (0)$

LÍMITE $x \to +\infty$
INDETER. $\left(\frac{\infty}{\infty}\right)$

LyC-4 Calcula el límite cuando $x \to +\infty$ de las siguientes expresiones:

a) $\lim_{x\to+\infty} \frac{3x+1}{\sqrt{4x^2-6x}}$

b) $\lim_{x\to+\infty} \frac{\sqrt{5x^3-2x}}{2x+5}$

c) $\lim_{x\to+\infty} \left[\sqrt{x^3-5x} - \frac{3x^2-7x+2}{2x-1}\right]$

d) $\lim_{x\to+\infty} \left[\frac{(x-5)^4}{x^2-1} - 1.5^x\right]$

e) $\lim_{x\to+\infty} \frac{e^x+\operatorname{sen} x}{e^x+\cos x}$

f) $\lim_{x\to\infty} \frac{e^x+8e^{2x}}{e^x+4e^{2x}}$

g) $\lim_{x\to+\infty} \left[\frac{\sqrt[3]{3+5x-8x^3}}{1+2x}\right]^{25}$

h) $\boxed{\lim_{x\to\infty} \frac{3x+2e^x}{7x+5e^x}}$

Solución:

a) $\lim_{x\to+\infty} \frac{3x+1}{\sqrt{4x^2-6x}} = \left(\frac{\infty}{\infty}\right) = \lim_{x\to+\infty} \frac{3x}{2x} = \frac{3}{2}$

b) $\lim_{x\to+\infty} \frac{\sqrt{5x^3-2x}}{2x+5} = \left(\frac{\infty}{\infty}\right) = \lim_{x\to+\infty} \frac{\sqrt{5}\cdot x^{3/2}}{2x} = +\infty$

c) $\lim_{x\to+\infty} \left(\sqrt{x^3-5x} - \frac{3x^2-7x+2}{2x-1}\right) = \left(\frac{\infty}{\infty}\right) = \lim_{x\to+\infty} \left(x^{3/2} - \frac{3x}{2}\right) = +\infty$

d) $\lim_{x\to+\infty} \left[\frac{(x-5)^4}{x^2-1} - 1.5^x\right] = \left(\frac{\infty}{\infty}\right) = -\infty$ Porque una expresión potencial es un infinito demayor orden que cualquier polinomio.

e) $\lim_{x\to+\infty} \frac{e^x+\operatorname{sen} x}{e^x+\cos x} = \left(\frac{\infty}{\infty}\right) = \lim_{x\to+\infty} \frac{1+\frac{\operatorname{sen} x}{e^x}}{1+\frac{\cos x}{e^x}} = 1$ Porque tanto el seno como el coseno son funciones acotadas entre -1 y 1

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$$\textbf{f)}\ \lim_{x\to\infty}\frac{e^x+8e^{2x}}{e^x+4e^{2x}}=\underbrace{\left(\frac{\infty}{\infty}\right)=\!=\!=\!=\!=}_{\text{Dividimos por } e^{2x}}\lim_{x\to\infty}\frac{1/e^x+8}{1/e^x+4}=\frac{8}{4}=2$$

$$\textbf{g)}\ \lim_{x\to+\infty}\left[\frac{\sqrt[3]{3+5x-8x^3}}{1+2x}\right]^{25}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to+\infty}\left[\frac{\sqrt[3]{\frac{3}{x^3}+\frac{5}{x^2}-8}}{\frac{1}{x}+2}\right]^{25}=\left[\frac{\sqrt[3]{-8}}{2}\right]^{25}=(-1)^{25}=-1$$

$$\boxed{\textbf{h)}\ \lim_{x\to\infty}\frac{3x+2e^x}{7x+5e^x}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{\frac{3x}{e^x}+2}{\frac{7x}{e^x}+5}=\frac{0+2}{0+5}=\frac{2}{5}}$$

LÍMITE $x \to +\infty$ INDETER. $(\infty - \infty)$

LyC-5 Calcula los siguientes límites.:

a) $\lim_{x\to+\infty}\left(\dfrac{2x^2-5x}{x+3}-2x\right)$

b) $\lim_{x\to+\infty}\left(\sqrt{x^2-x}-x\right)$

c) $\lim_{x\to+\infty}\left(\dfrac{3x^3+5}{x+2}-\dfrac{4x^3-x}{x-2}\right)$

d) $\lim_{x\to+\infty}\left(\dfrac{x^3}{2x^2+1}-\dfrac{x}{2}\right)$

e) $\lim_{x\to+\infty}\left(\dfrac{3x+5}{2}-\dfrac{x^2-2}{x}\right)$

f) $\lim_{x\to+\infty}\left(\sqrt{x^2+x}-\sqrt{x^2+1}\right)$

g) $\lim_{x\to+\infty}\left(2x-\sqrt{x^2+x}\right)$

h) $\lim_{x\to+\infty}\left(\sqrt{x+1}-\sqrt{x+2}\right)$

i) $\lim_{x\to+\infty}\left(\dfrac{3x^3+5}{x+2}-\dfrac{4x^3-x}{x-2}\right)$

j) $\lim_{x\to+\infty}\left(\dfrac{x^3}{2x^2+1}-\dfrac{x}{2}\right)$

Solución:

a) $$\lim_{x\to+\infty}\left(\frac{2x^2-5x}{x+3}-2x\right)=(\infty-\infty)=\lim_{x\to+\infty}\frac{2x^2-5x-2x\cdot(x+3)}{x+3}=\lim_{x\to+\infty}\frac{-11x}{x+3}=-11$$

b) $$\lim_{x\to+\infty}\left(\sqrt{x^2-x}-x\right)=(\infty-\infty)=\lim_{x\to+\infty}\frac{\left(\sqrt{x^2-x}-x\right)\cdot\left(\sqrt{x^2-x}+x\right)}{\sqrt{x^2-x}+x}=\lim_{x\to+\infty}\frac{\left(x^2-x\right)-x^2}{\sqrt{x^2-x}+x}=$$

$$=\lim_{x\to+\infty}\frac{-x}{\sqrt{x^2-x}+x}=\lim_{x\to+\infty}\frac{-x}{x+x}=-\frac{1}{2}$$

c) $$\lim_{x\to+\infty}\left(\frac{3x^3+5}{x+2}-\frac{4x^3-x}{x-2}\right)=(\infty-\infty)=\lim_{x\to+\infty}\left(3x^2-4x^2\right)=-\infty$$

d) $\lim\limits_{x\to+\infty}\left(\dfrac{x^3}{2x^2+1}-\dfrac{x}{2}\right)=(\infty-\infty)=\lim\limits_{x\to+\infty}\dfrac{2x^3-x\cdot\left(2x^2+1\right)}{4x^2+2}=\lim\limits_{x\to+\infty}\dfrac{-x}{4x^2+2}=0$

e) $\lim\limits_{x\to+\infty}\left(\dfrac{3x+5}{2}-\dfrac{x^2-2}{x}\right)=(\infty-\infty)=\left\langle\begin{array}{l}=\lim\limits_{x\to+\infty}\left(\dfrac{3}{2}x-x\right)=\\[2ex]=\lim\limits_{x\to+\infty}\dfrac{=3x^2+5x-\left(2x^2-4\right)}{2x}=\lim\limits_{x\to+\infty}\dfrac{x^2+5x+4}{2x}\end{array}\right\rangle=+\infty$

f) $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+x}-\sqrt{x^2+1}\right)=(\infty-\infty)=\lim\limits_{x\to+\infty}\dfrac{\left(\sqrt{x^2+x}-\sqrt{x^2+1}\right)\cdot\left(\sqrt{x^2+x}+\sqrt{x^2+1}\right)}{\sqrt{x^2+x}+\sqrt{x^2+1}}=$

$=\lim\limits_{x\to+\infty}\dfrac{\cancel{x^2}+x-\left(\cancel{x^2}+1\right)}{\sqrt{x^2+x}+\sqrt{x^2+1}}=\lim\limits_{x\to+\infty}\dfrac{x-1}{\sqrt{x^2+x}+\sqrt{x^2+1}}=\lim\limits_{x\to+\infty}\dfrac{x}{x+x}=\dfrac{1}{2}$

g) $\lim\limits_{x\to+\infty}\left(2x-\sqrt{x^2+x}\right)=(\infty-\infty)=\lim\limits_{x\to+\infty}\dfrac{\left(2x-\sqrt{x^2+x}\right)\cdot\left(2x+\sqrt{x^2+x}\right)}{2x+\sqrt{x^2+x}}=\lim\limits_{x\to+\infty}\dfrac{4x^2-\left(x^2+x\right)}{2x+\sqrt{x^2+x}}=$

$=\lim\limits_{x\to+\infty}\dfrac{3x^2-x}{2x+\sqrt{x^2+x}}=\lim\limits_{x\to+\infty}\dfrac{3x^2}{2x+x}=+\infty$

h) $\lim\limits_{x\to+\infty}\left(\sqrt{x+1}-\sqrt{x+2}\right)=(\infty-\infty)=\lim\limits_{x\to+\infty}\dfrac{\left(\sqrt{x+1}-\sqrt{x+2}\right)\cdot\left(\sqrt{x+1}+\sqrt{x+2}\right)}{\sqrt{x+1}+\sqrt{x+2}}=$

$=\lim\limits_{x\to+\infty}\dfrac{x+1-(x+2)}{\sqrt{x+1}+\sqrt{x+2}}=\lim\limits_{x\to+\infty}\dfrac{-1}{\sqrt{x+1}+\sqrt{x+2}}=0$

i) $\lim\limits_{x\to+\infty}\left(\dfrac{3x^3+5}{x+2}-\dfrac{4x^3-x}{x-2}\right)=(\infty-\infty)=\lim\limits_{x\to+\infty}\dfrac{\left(3x^3+5\right)\cdot(x-2)-\left(4x^3-x\right)\cdot(x+2)}{(x+2)\cdot(x-2)}=$

$=\lim\limits_{x\to+\infty}\dfrac{\left(3x^4-6x^3+5x-10\right)-\left(4x^4+8x^3-x^2-2x\right)}{(x+2)\cdot(x-2)}=$

$=\lim\limits_{x\to+\infty}\dfrac{-x^4-14x^3+x^2+7x-10}{x^2-4}=-\infty$

j) $\lim\limits_{x\to+\infty}\left(\dfrac{x^3}{2x^2+1}-\dfrac{x}{2}\right)=(\infty-\infty)=\lim\limits_{x\to+\infty}\dfrac{\cancel{2x^3}-\cancel{2x^3}-x}{4x^2+2}=\lim\limits_{x\to+\infty}\dfrac{-x}{4x^2+2}=0$

LÍMITE $x \to +\infty$ INDETER. $\left(k^{\infty}\right)$

LyC-6 Halla los siguientes límites cuando $x \to +\infty$.

a) $\lim\limits_{x\to+\infty}\left(1+\frac{1}{5x}\right)^{x}$ b) $\lim\limits_{x\to+\infty}\left(5+\frac{1}{5x}\right)^{5x}$ c) $\lim\limits_{x\to+\infty}\left(1+\frac{1}{5x}\right)^{5}$

d) $\lim\limits_{x\to+\infty}\left(1+\frac{5}{5x}\right)^{x}$ e) $\lim\limits_{x\to+\infty}\left(1+\frac{1}{x}\right)^{3x-2}$ f) $\lim\limits_{x\to+\infty}\left(1-\frac{1}{2x}\right)^{4x}$

g) $\lim\limits_{x\to+\infty}\left(1+\frac{1}{5x}\right)^{3x}$ h) $\lim\limits_{x\to+\infty}\left(1+\frac{3}{2x}\right)^{5}$ i) $\lim\limits_{x\to+\infty}\left(1-\frac{1}{2x}\right)^{3x}$

j) $\lim\limits_{x\to+\infty}\left(1+\frac{2}{5x}\right)^{5x}$ k) $\lim\limits_{x\to+\infty}\left(\frac{x^2+x-1}{x^2+2}\right)^{3x-1}$ l) $\lim\limits_{x\to+\infty}\left(1+\frac{1}{x}\right)^{3x-1}$

m) $\lim\limits_{x\to+\infty}\left(1+\frac{x^2-1}{x^2+6}\right)^{3x-2}$ n) $\boxed{\lim\limits_{x\to 0}\left(1+4x\right)^{32/x^3}}$ o) $\boxed{\lim\limits_{x\to 0}\left(1+\operatorname{arctg} x\right)^{a/x}}$

p) $\boxed{\lim\limits_{x\to+\infty}\left(\frac{x^2-3}{x^2+3}\right)^{ax^2}}$

Solución:

a) $\lim\limits_{x\to+\infty}\left(1+\frac{1}{5x}\right)^{x}=\left(1^{\infty}\right)=e^{\lim\limits_{x\to+\infty} x\cdot\left(1+\frac{1}{5x}-1\right)}=e^{\lim\limits_{x\to+\infty}\frac{x}{5x}}=e^{\frac{1}{5}}$

b) $\lim\limits_{x\to+\infty}\left(5+\frac{1}{5x}\right)^{5x}=5^{(+\infty)}=+\infty$

c) $\lim\limits_{x\to+\infty}\left(1+\frac{1}{5x}\right)^{5}=1^{5}=1$

$$\textbf{d)}\ \lim_{x\to+\infty}\left(1+\frac{5}{5x}\right)^{x}=\left(1^{\infty}\right)=e^{\lim\limits_{x\to+\infty} x\cdot\left(1+\frac{5}{5x}-1\right)}=e^{\lim\limits_{x\to+\infty}\frac{5x}{5x}}=e$$

$$\textbf{e)}\ \lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^{3x-2}=\left(1^{\infty}\right)=e^{\lim\limits_{x\to+\infty}(3x-2)\cdot\left(1+\frac{1}{x}-1\right)}=e^{\lim\limits_{x\to+\infty}\frac{3x-2}{x}}=e^{3}$$

$$\textbf{f)}\ \lim_{x\to+\infty}\left(1-\frac{1}{2x}\right)^{4x}=\left(1^{\infty}\right)=e^{\lim\limits_{x\to+\infty}4x\cdot\left(1-\frac{1}{2x}-1\right)}=e^{\lim\limits_{x\to+\infty}\frac{-4x}{2x}}=e^{-2}$$

$$\textbf{g)}\ \lim_{x\to+\infty}\left(1+\frac{1}{5x}\right)^{3x}=\left(1^{\infty}\right)=e^{\lim\limits_{x\to+\infty}3x\cdot\left(\cancel{1}+\frac{1}{5x}-\cancel{1}\right)}=e^{\lim\limits_{x\to+\infty}\frac{3x}{5x}}=e^{\frac{3}{5}}$$

$$\textbf{h)}\ \lim_{x\to+\infty}\left(1+\frac{3}{2x}\right)^{5}=1^{5}=1$$

$$\textbf{i)}\ \lim_{x\to+\infty}\left(1-\frac{1}{2x}\right)^{3x}=\left(1^{\infty}\right)=e^{\lim\limits_{x\to+\infty}3x\cdot\left(\cancel{1}-\frac{1}{2x}-\cancel{1}\right)}=e^{\lim\limits_{x\to+\infty}\frac{-3x}{2x}}=e^{\frac{-3}{2}}$$

$$\textbf{j)}\ \lim_{x\to+\infty}\left(1+\frac{2}{5x}\right)^{5x}=\left(1^{\infty}\right)=e^{\lim\limits_{x\to+\infty}5x\cdot\left(\cancel{1}+\frac{2}{5x}-\cancel{1}\right)}=e^{\lim\limits_{x\to+\infty}\frac{10x}{5x}}=e^{2}$$

$$\textbf{k)}\ \lim_{x\to+\infty}\left(\frac{x^{2}+x-1}{x^{2}+2}\right)^{3x-1}=\left(1^{\infty}\right)=e^{\lim\limits_{x\to+\infty}(3x-1)\cdot\left(\frac{x^{2}+x-1}{x^{2}+2}-1\right)}=e^{\lim\limits_{x\to+\infty}(3x-1)\cdot\left(\frac{\cancel{x^{2}}+x-1-\left(\cancel{x^{2}}+2\right)}{x^{2}+2}\right)}=e^{\lim\limits_{x\to+\infty}(3x-1)\cdot\left(\frac{x-3}{x^{2}+2}\right)}=$$

$$=e^{\lim\limits_{x\to+\infty}\left(\frac{3x^{2}-10x+3}{x^{2}+2}\right)}=e^{3}$$

$$\textbf{l)}\ \lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^{3x-1}=\left(1^{\infty}\right)=e^{\lim\limits_{x\to+\infty}(3x-1)\cdot\left(\cancel{1}+\frac{1}{x}-\cancel{1}\right)}=e^{\lim\limits_{x\to+\infty}\frac{3x-1}{x}}=e^{3}$$

$$\textbf{m)}\ \lim_{x\to+\infty}\left(1+\frac{x^{2}-1}{x^{2}+6}\right)^{3x-2}=\left(2^{+\infty}\right)=+\infty$$

$$\boxed{\textbf{n)}\ \lim_{x\to0}\left(1+4x\right)^{32/x^{3}}=\left(1^{\infty}\right)=e^{\lim\limits_{x\to0}\frac{32}{x^{3}}\cdot(1+4x-1)}=e^{\lim\limits_{x\to0}\frac{128\cancel{x}}{x^{\cancel{3}2}}}=\left(e^{k/0}\right)\overset{\overbrace{\text{Como el denominador es una potencia par los limites laterales serán siempre iguales a }+\infty}}{=\!=\!=\!=\!=\!=}\left(e^{+\infty}\right)=+\infty}$$

o) $\lim\limits_{x\to 0}(1+\operatorname{arctg} x)^{a/x} = (1^{\infty}) = e^{\lim\limits_{x\to 0}\frac{a}{x}\cdot(1+\operatorname{arctg} x-1)} = e^{\lim\limits_{x\to 0}\frac{a\cdot\operatorname{arctg} x}{x}} = e^{\left(\frac{0}{0}\right)} \overset{\overbrace{L'Hôpital}}{=\!=\!=\!=\!=} e^{\lim\limits_{x\to 0}\frac{a\cdot\frac{1}{1+x^2}}{1}} = e^{\lim\limits_{x\to 0}\frac{a}{1+x^2}} = e^{a}$

p) $\lim\limits_{x\to+\infty}\left(\frac{x^2-3}{x^2+3}\right)^{ax^2} = (1^{\infty}) = e^{\lim\limits_{x\to+\infty} ax^2\cdot\left(\frac{x^2-3}{x^2+3}-1\right)} = e^{\lim\limits_{x\to+\infty} ax^2\cdot\left(\frac{\cancel{x^2}-3-\cancel{x^2}-3}{x^2+3}\right)} = e^{\lim\limits_{x\to+\infty}\frac{-6ax^2}{x^2+3}} = e^{-6a} = \frac{1}{e^{6a}}$

LÍMITE $x \to -\infty$

INDETER. $(\infty - \infty)$

LyC-7 Hallar el límite cuando $x \to -\infty$ de las siguientes expresiones:

a) $\lim\limits_{x\to-\infty} \dfrac{-3x^3-5x^2+3}{6x^3-7x^2}$

b) $\lim\limits_{x\to-\infty} \dfrac{\sqrt{5x^3-2x}}{2x+5}$

c) $\lim\limits_{x\to-\infty} \left[\sqrt{-3x^3+5}+\dfrac{4x^2-3}{x+2}\right]$

d) $\lim\limits_{x\to-\infty} \left[\sqrt{x^2-x}-x\right]$

e) $\lim\limits_{x\to-\infty} \left[\sqrt{x^2+x}+x\right]$

f) $\lim\limits_{x\to-\infty} \left(x^2-\sqrt[3]{2x+1}\right)$

g) $\lim\limits_{x\to-\infty} \left(x^2+2^x\right)$

h) $\lim\limits_{x\to-\infty} \left(x^2-2^x\right)$

i) $\lim\limits_{x\to-\infty} \left(x^2-2^{-x}\right)$

j) $\lim\limits_{x\to-\infty} \left(2^{-x}-3^{-x}\right)$

k) $\lim\limits_{x\to-\infty} \left(\sqrt{x^5-1}-5^x\right)$

l) $\lim\limits_{x\to-\infty} \left(2^x-x^2\right)$

m) $\lim\limits_{x\to-\infty} \left(x^2-\sqrt{x^4-1}\right)$

n) $\lim\limits_{x\to-\infty} \left(\sqrt[3]{x+2}-x^2\right)$

o) $\lim\limits_{x\to-\infty} \left(3^{-x}-2^{-x}\right)$

p) $\lim\limits_{x\to-\infty} \dfrac{3x^3+5}{x+2}-\dfrac{x^3-x}{x-2}$

q) $\lim\limits_{x\to-\infty} \left(\dfrac{x^3}{2x^2+1}-\dfrac{x}{2}\right)$

r) $\lim\limits_{x\to-\infty} \left(\sqrt{x^2+x}-\sqrt{x^2+1}\right)$

s) $\lim\limits_{x\to-\infty} \left(2x+\sqrt{x^2+x}\right)$

t) $\lim\limits_{x\to-\infty} \left(\sqrt{x^2+2x}+x\right)$

u) $\lim\limits_{x\to-\infty} \left[\sqrt{-3x^3+5}-\dfrac{4x^2-3}{x+2}\right]$

v) $\lim\limits_{x\to 0} \left(\dfrac{1}{\ln(1+x)}-\dfrac{1}{x}\right)$

Solución:

a) $\lim\limits_{x\to-\infty} \dfrac{-3x^3-5x^2+3}{6x^3-7x^2} = \lim\limits_{x\to+\infty} \dfrac{-3(-x)^3}{6(-x)^3} = \lim\limits_{x\to+\infty} \dfrac{3x^3}{-6x^3} = \left(\dfrac{\infty}{\infty}\right) = -\dfrac{1}{2}$

b) $\lim\limits_{x\to-\infty}\dfrac{\sqrt{5x^3-2x}}{2x+5}=\lim\limits_{x\to+\infty}\dfrac{\sqrt{5(-x)^3}}{2(-x)}=\lim\limits_{x\to+\infty}\dfrac{\sqrt{-5x^3}}{-2x}=\nexists\left(\textit{raíz de índice par de nº negativo}\right)$

c) $\lim\limits_{x\to-\infty}\left[\sqrt{-3x^3+5}+\dfrac{4x^2-3}{x+2}\right]=\lim\limits_{x\to+\infty}\left[\sqrt{-3(-x)^3+5}+\dfrac{4(-x)^2-3}{(-x)+2}\right]=$

$$=\lim_{x\to+\infty}\left[\sqrt{3x^3+5}+\frac{4x^2-3}{-x+2}\right]=\lim_{x\to+\infty}\sqrt{3}x^{3/2}-4x=+\infty$$

d) $\lim\limits_{x\to-\infty}\left[\sqrt{x^2-x}-x\right]=\lim\limits_{x\to+\infty}\left[\sqrt{(-x)^2-(-x)}-(-x)\right]=\lim\limits_{x\to+\infty}\left[\sqrt{x^2+x}+x\right]=(+\infty)+(+\infty)=+\infty$

e) $\lim\limits_{x\to-\infty}\left[\sqrt{x^2+x}+x\right]=\lim\limits_{x\to+\infty}\left[\sqrt{(-x)^2+(-x)}+(-x)\right]=\lim\limits_{x\to+\infty}\left[\sqrt{x^2-x}-x\right]=(\infty-\infty)=$

$$=\lim_{x\to+\infty}\frac{\left(\sqrt{x^2-x}-x\right)\cdot\left(\sqrt{x^2-x}+x\right)}{\sqrt{x^2-x}+x}=\lim_{x\to+\infty}\frac{\cancel{x^2}-x-\cancel{x^2}}{\sqrt{x^2-x}+x}=$$

$$=\lim_{x\to+\infty}\frac{-x}{\sqrt{x^2-x}+x}=-\frac{1}{1+1}=-\frac{1}{2}$$

f) $\lim\limits_{x\to-\infty}\left(x^2-\sqrt[3]{2x+1}\right)=\lim\limits_{x\to+\infty}\left[(-x)^2-\sqrt[3]{2(-x)+1}\right]=\lim\limits_{x\to+\infty}\left[x^2-\sqrt[3]{-2x+1}\right]=+\infty$

g) $\lim\limits_{x\to-\infty}\left(x^2+2^x\right)=\lim\limits_{x\to+\infty}\left[(-x)^2+2^{-x}\right]=\lim\limits_{x\to+\infty}\left(x^2+\dfrac{1}{2^x}\right)=+\infty$

h) $\lim\limits_{x\to-\infty}\left(x^2-2^x\right)=\lim\limits_{x\to+\infty}\left[(-x)^2-2^{-x}\right]=\lim\limits_{x\to+\infty}\left[x^2-\dfrac{1}{2^x}\right]=+\infty$

i) $\lim\limits_{x\to-\infty}\left(x^2-2^{-x}\right)=\lim\limits_{x\to+\infty}\left[(-x)^2-2^{-(-x)}\right]=\lim\limits_{x\to+\infty}\left(x^2-2^x\right)=-\infty$

j) $\lim\limits_{x\to-\infty}\left(2^{-x}-3^{-x}\right)=\lim\limits_{x\to+\infty}\left[2^{-(-x)}-3^{-(-x)}\right]=\lim\limits_{x\to+\infty}\left(2^x-3^x\right)=-\infty$

k) $\lim\limits_{x\to-\infty}\left(\sqrt{x^5-1}-5^x\right)=\lim\limits_{x\to+\infty}\left[\sqrt{(-x)^5-1}-5^{-x}\right]=\lim\limits_{x\to+\infty}\left[\sqrt{-x^5-1}-\dfrac{1}{5^x}\right]=\textit{No existe}$

l) $\lim\limits_{x\to-\infty}\left(2^x-x^2\right)=\lim\limits_{x\to+\infty}\left(2^{-x}-(-x)^2\right)=\lim\limits_{x\to+\infty}\left(\dfrac{1}{2^x}-x^2\right)=-\infty$

m) $\lim\limits_{x\to-\infty}\left(x^2-\sqrt{x^4-1}\right)=\lim\limits_{x\to+\infty}\left[(-x)^2-\sqrt{(-x)^4-1}\right]=\lim\limits_{x\to+\infty}\left(x^2-\sqrt{x^4-1}\right)=$

$$=(\infty-\infty)=\lim_{x\to+\infty}\frac{\left(x^2-\sqrt{x^4-1}\right)\cdot\left(x^2+\sqrt{x^4-1}\right)}{x^2+\sqrt{x^4-1}}=\lim_{x\to+\infty}\frac{\cancel{x^4}-\left(\cancel{x^4}-1\right)}{x^2+\sqrt{x^4-1}}=0$$

n) $\lim\limits_{x\to-\infty}\left(\sqrt[3]{x+2}-x^2\right)=\lim\limits_{x\to-\infty}\left(x^{1/3}-x^2\right)=-\infty$

o) $\lim\limits_{x\to-\infty}\left(3^{-x}-2^{-x}\right)=\lim\limits_{x\to+\infty}\left[3^{-(-x)}-2^{-(-x)}\right]=\lim\limits_{x\to+\infty}\left(3^x-2^x\right)=+\infty$

p) $\lim\limits_{x\to-\infty}\left(\dfrac{3x^3+5}{x+2}-\dfrac{x^3-x}{x-2}\right)=\lim\limits_{x\to+\infty}\left(\dfrac{3\cdot(-x)^3+5}{(-x)+2}-\dfrac{(-x)^3-(-x)}{(-x)-2}\right)=\lim\limits_{x\to+\infty}\left(\dfrac{-3x^3+5}{-x+2}-\dfrac{-x^3+x}{-x-2}\right)=$

$$=(\infty-\infty)=\lim_{x\to+\infty}\frac{(2+x)\cdot\left(-3x^3+5\right)+(2-x)\cdot\left(-x^3+x\right)}{(2+x)\cdot(2-x)}=$$

$$=\lim_{x\to+\infty}\frac{-3x^4-6x^3+5x+10+x^4-2x^3-x^2+2x}{4-x^2}=$$

$$=\lim_{x\to-\infty}\frac{-2x^4-8x^3-x^2+7x+10}{4-x^2}=+\infty$$

q) $\lim\limits_{x\to-\infty}\left(\dfrac{x^3}{2x^2+1}-\dfrac{x}{2}\right)=\lim\limits_{x\to+\infty}\left(\dfrac{(-x)^3}{2\cdot(-x)^2+1}-\dfrac{(-x)}{2}\right)=(\infty-\infty)=\lim\limits_{x\to+\infty}\left(\dfrac{-x^3}{2x^2+1}+\dfrac{x}{2}\right)=$

$$=\lim_{x\to+\infty}\left(\frac{\cancel{-2x^3}+\cancel{2x^3}+x}{4x^2+2}\right)=\lim_{x\to+\infty}\left(\frac{x}{4x^2+2}\right)=0$$

r) $\lim\limits_{x\to-\infty}\left(\sqrt{x^2+x}-\sqrt{x^2+1}\right)=\lim\limits_{x\to+\infty}\left(\sqrt{(-x)^2+(-x)}-\sqrt{(-x)^2+1}\right)=\lim\limits_{x\to+\infty}\left(\sqrt{x^2-x}-\sqrt{x^2+1}\right)=$

$$=(\infty-\infty)=\lim_{x\to+\infty}\frac{\left(\sqrt{x^2-x}-\sqrt{x^2+1}\right)\cdot\left(\sqrt{x^2-x}+\sqrt{x^2+1}\right)}{\sqrt{x^2-x}+\sqrt{x^2+1}}=$$

$$=\lim_{x\to+\infty}\frac{\cancel{x^2}-x-\left(\cancel{x^2}+1\right)}{\sqrt{x^2-x}+\sqrt{x^2+1}}==\lim_{x\to+\infty}\frac{-x-1}{\sqrt{x^2-x}+\sqrt{x^2+1}}=\lim_{x\to+\infty}\frac{-x-1}{2x}=-\frac{1}{2}$$

s) $\lim\limits_{x\to-\infty}\left(2x+\sqrt{x^2+x}\right)=\lim\limits_{x\to+\infty}\left(2\cdot(-x)+\sqrt{(-x)^2+(-x)}\right)=\lim\limits_{x\to+\infty}\left(-2x+\sqrt{x^2-x}\right)=(\infty-\infty)=$

$$=\lim_{x\to+\infty}\frac{\left(-2x+\sqrt{x^2-x}\right)\cdot\left(-2x-\sqrt{x^2-x}\right)}{-2x-\sqrt{x^2-x}}=\lim_{x\to+\infty}\frac{4x^2+x^2-x}{-2x-\sqrt{x^2-x}}=-\infty$$

$$\textbf{t)}\ \lim_{x\to-\infty}\left(\sqrt{x^2+2x}+x\right)=\lim_{x\to+\infty}\left(\sqrt{(-x)^2+2\cdot(-x)}+(-x)\right)=\lim_{x\to+\infty}\left(\sqrt{x^2-2x}-x\right)=(\infty-\infty)=$$

$$=\lim_{x\to+\infty}\frac{\left(\sqrt{x^2-2x}-x\right)\cdot\left(\sqrt{x^2-2x}+x\right)}{\sqrt{x^2-2x}+x}=\lim_{x\to+\infty}\frac{\cancel{x^2}-2x-\cancel{x^2}}{\sqrt{x^2-2x}+x}=\lim_{x\to+\infty}\frac{-2x}{2x}=-1$$

$$\textbf{u)}\ \lim_{x\to-\infty}\left[\sqrt{-3x^3+5}-\frac{4x^2-3}{x+2}\right]=\lim_{x\to+\infty}\left[\sqrt{-3(-x)^3+5}-\frac{4(-x)^2-3}{(-x)+2}\right]=\lim_{x\to+\infty}\left[\sqrt{3x^3+5}-\frac{4x^2-3}{-x+2}\right]=$$

$$=\lim_{x\to+\infty}\left(\sqrt{3}x^{3/2}+2x\right)=(+\infty)+(+\infty)=+\infty$$

$$\textbf{v)}\ \lim_{x\to0}\left(\frac{1}{\ln(1+x)}-\frac{1}{x}\right)=\lim_{x\to0}\left(\frac{1}{\ln(1+x)}-\frac{1}{x}\right)=(\infty-\infty)=\lim_{x\to0}\frac{x-\ln(1+x)}{x\cdot\ln(1+x)}=\left(\frac{0}{0}\right)\overbrace{=\!=\!=\!=\!=}^{L'H\hat{o}pital}$$

$$=\lim_{x\to0}\frac{1-\dfrac{1}{1+x}}{\ln(1+x)+\dfrac{x}{1+x}}=\lim_{x\to0}\frac{\dfrac{1+x-1}{\cancel{1+x}}}{\dfrac{(1+x)\cdot\ln(1+x)+x}{\cancel{1+x}}}=\lim_{x\to0}\frac{x}{(1+x)\cdot\ln(1+x)+x}=$$

$$=\left(\frac{0}{0}\right)\overbrace{=\!=\!=\!=\!=}^{L'H\hat{o}pital}\lim_{x\to0}\frac{1}{\ln(1+x)+\dfrac{1+x}{1+x}+1}=\lim_{x\to0}\frac{1}{\ln(1+x)+2}=\frac{1}{2}$$

LÍMITE $x \to -\infty$ INDETER. $\left(k^{\infty}\right)$

LyC-8 Calcula el límite cuando $x \to -\infty$ de las siguientes expresiones:

a) $\lim\limits_{x\to-\infty}\left(1+\dfrac{3}{x}\right)^{2x}$ b) $\lim\limits_{x\to-\infty}\left(1-\dfrac{1}{x}\right)^{5x+3}$

c) $\lim\limits_{x\to-\infty}\left(\dfrac{x^2+x-1}{x^2+2}\right)^{3x-1}$ d) $\text{lím}\limits_{x\to-\infty}\left(1+\dfrac{3}{2x}\right)^{5x}$

e) $\lim\limits_{x\to\pi/2}(1+2\cos x)^{1/\cos x}$

Solución:

a) $\text{lím}\limits_{x\to-\infty}\left(1+\dfrac{3}{x}\right)^{2x} = \text{lím}\limits_{x\to+\infty}\left(1+\dfrac{3}{(-x)}\right)^{2\cdot(-x)} = \text{lím}\limits_{x\to+\infty}\left(1-\dfrac{3}{x}\right)^{-2x} = \left(1^{\infty}\right) = e^{\text{lím}\limits_{x\to+\infty}(-2x)\cdot\left(1-\frac{3}{x}-1\right)} = e^{\text{lím}\limits_{x\to+\infty}\frac{6x}{x}} = e^{6}$

b) $\text{lím}\limits_{x\to-\infty}\left(1-\dfrac{1}{x}\right)^{5x+3} = \text{lím}\limits_{x\to+\infty}\left(1-\dfrac{1}{(-x)}\right)^{5\cdot(-x)+3} = \text{lím}\limits_{x\to+\infty}\left(1+\dfrac{1}{x}\right)^{-5x+3} = \left(1^{\infty}\right) = e^{\text{lím}\limits_{x\to+\infty}(-5x+3)\cdot\left(1+\frac{1}{x}-1\right)} =$

$= e^{\text{lím}\limits_{x\to+\infty}\frac{-5x}{x}} = e^{-5}$

c) $\text{lím}\limits_{x\to-\infty}\left(\dfrac{x^2+x-1}{x^2+2}\right)^{3x-1} = \text{lím}\limits_{x\to+\infty}\left(\dfrac{(-x)^2-(-x)-1}{(-x)^2+2}\right)^{3(-x)-1} = \text{lím}\limits_{x\to+\infty}\left(\dfrac{x^2+x-1}{x^2+2}\right)^{-3x-1} = \left(1^{-\infty}\right) = 0$

d) $\text{lím}\limits_{x\to-\infty}\left(1+\dfrac{3}{2x}\right)^{5x} = \text{lím}\limits_{x\to+\infty}\left(1+\dfrac{3}{2(-x)}\right)^{5\cdot(-x)} = \text{lím}\limits_{x\to+\infty}\left(1-\dfrac{3}{2x}\right)^{-5x} = \left(1^{\infty}\right) =$

$= e^{\text{lím}\limits_{x\to+\infty}(-5x)\cdot\left(1-\frac{3}{2x}-1\right)} = e^{\text{lím}\limits_{x\to+\infty}\frac{15x}{2x}} = e^{15/2}$

e) $\lim\limits_{x\to\pi/2}(1+2\cos x)^{1/\cos x}=\left(1^{\infty}\right)=e^{\lim\limits_{x\to\pi/2}\frac{1}{\cos x}\cdot(1+2\cos x-1)}=e^{\lim\limits_{x\to\pi/2}\frac{2\cos x}{\cos x}}=e^{\lim\limits_{x\to\pi/2}\frac{2\cos x}{\cos x}}=e^{2}$

LÍMITE $x \to k$

INDETER. $\left(\frac{0}{0}\right)$ y $\left(\frac{k}{0}\right)$

LyC-9 Calcula los siguientes límites:

a) $\lim\limits_{x \to 3} \left[\dfrac{5}{(x-3)^2} + 2^{1/(x-3)^2} \right]$

b) $\lim\limits_{x \to -2} \dfrac{x^2-4}{x^3+2x^2+5x+10}$

c) $\lim\limits_{x \to 1} \dfrac{x^2+3}{x^2-5x+4}$

d) $\lim\limits_{x \to 2} \dfrac{\sqrt[3]{x^2-2x}}{\sqrt{x^2+x-6}} = \dfrac{(0)}{(0)}$

e) $\lim\limits_{x \to -1} \dfrac{x^3-2x^2+2x+5}{x^2-6x-7}$

f) $\lim\limits_{x \to -3} \dfrac{\sqrt{x^2+2x-3}}{\sqrt[3]{x^3+3x^2}}$

g) $\lim\limits_{x \to 1} \dfrac{\sqrt[4]{x^3-x}}{\sqrt{x^2+x-2}}$

h) $\lim\limits_{x \to 6} \left(\dfrac{x^2-4x-10}{x-4} \right)^{\frac{1}{x-6}}$

i) $\lim\limits_{x \to \frac{3\pi}{2}} (1+\operatorname{sen} x) \cdot \operatorname{tg}^2 x$

j) $\lim\limits_{x \to 0} \dfrac{\sqrt{1-\cos x}}{x}$

k) $\lim\limits_{x \to 0} \dfrac{\operatorname{sen} x}{x}$

l) $\lim\limits_{x \to 0} \dfrac{1-\cos x}{x^2}$

m) $\lim\limits_{x \to -1} \dfrac{\sqrt{2+x}-1}{\sqrt{8-x}-3}$

Solución:

a) $\lim\limits_{x \to 3} \left[\dfrac{5}{(x-3)^2} + 2^{1/(x-3)^2} \right] = \dfrac{5}{(0)} + 2^{1/(0)} = (+\infty) + (+\infty) = +\infty$ $\left\{ \begin{array}{l} \text{No hay que hacer límites} \\ \text{laterales porque los } (0) \\ \text{están al cuadrado} \end{array} \right\}$

b) $\lim\limits_{x \to -2} \dfrac{x^2-4}{x^3+2x^2+5x+10} = \left(\dfrac{0}{0}\right) = \lim\limits_{x \to -2} \dfrac{\cancel{(x+2)} \cdot (x-2)}{\cancel{(x+2)} \cdot (x^2+5)} = \lim\limits_{x \to -2} \dfrac{x-2}{x^2+5} = \dfrac{-4}{9}$

$$\textbf{c)}\lim_{x\to1}\frac{x^2+3}{x^2-5x+4}=\lim_{x\to1^+}\frac{x^2+3}{(x-4)\cdot(x-1)}=\left(\frac{4}{0}\right)=\{\text{límites laterales}\}=\left\langle\begin{array}{l}\lim\limits_{x\to1^+}\dfrac{x^2+3}{(x-4)\cdot(x-1)}=-\infty\\ \lim\limits_{x\to1^-}\dfrac{x^2+3}{(x-4)\cdot(x-1)}=+\infty\end{array}\right.$$

$$\textbf{d)}\lim_{x\to2}\frac{\sqrt[3]{x^2-2x}}{\sqrt{x^2+x-6}}=\frac{(0)}{(0)}=\left\{\begin{array}{l}\text{factorizamos y reducimos}\\ \text{las raíces al mismo exponente}\end{array}\right\}=\lim_{x\to2}\frac{\sqrt[3]{x\cdot(x-2)}}{\sqrt{(x-2)\cdot(x+3)}}=$$

$$=\lim_{x\to2}\frac{\sqrt[6]{x^2\cdot\cancel{(x-2)^2}}}{\sqrt[6]{(x-2)^{\cancel{3}}\cdot(x+3)^3}}=\lim_{x\to2}\sqrt[6]{\frac{x^2}{(x-2)\cdot(x+3)^3}}=\left(\frac{k}{0}\right)=$$

$$=\left\langle\begin{array}{l}\lim\limits_{x\to2^+}\sqrt[6]{\dfrac{x^2}{(x-2)\cdot(x+3)^3}}=+\infty\\ \lim\limits_{x\to2^-}\sqrt[6]{\dfrac{x^2}{(x-2)\cdot(x+3)^3}}=\text{No existe}\end{array}\right.$$

$$\textbf{e)}\lim_{x\to-1}\frac{x^3-2x^2+2x+5}{x^2-6x-7}=\left(\frac{0}{0}\right)=\left\langle\begin{array}{l}=\lim\limits_{x\to-1}\dfrac{\cancel{(x+1)}\cdot(x^2-3x+5)}{\cancel{(x+1)}\cdot(x-7)}=\lim\limits_{x\to-1}\dfrac{x^2-3x+5}{x-7}\\ =\{\text{L'Hôpital}\}=\lim\limits_{x\to-1}\dfrac{3x^2-4x+2}{2x-6}\end{array}\right\rangle=-\frac{9}{8}$$

	1	-2	2	5
-1		-1	3	-5
	1	-3	5	0

$$x=\frac{6\pm\sqrt{36+28}}{2}=\frac{6\pm8}{2}=\left\langle\begin{array}{l}x=-1\\ x=7\end{array}\right.$$

$$\textbf{f)}\lim_{x\to-3}\frac{\sqrt{x^2+2x-3}}{\sqrt[3]{x^3+3x^2}}=\left(\frac{0}{0}\right)=\lim_{x\to-3}\frac{\sqrt{(x-1)\cdot(x+3)}}{\sqrt[3]{x^2\cdot(x+3)}}=\lim_{x\to-3}\sqrt[6]{\frac{(x-1)^3\cdot(x+3)^{\cancel{3}}}{x^4\cdot\cancel{(x+3)^2}}}=$$

$$=\lim_{x\to-3}\sqrt[6]{\frac{(x-1)^3\cdot(x+3)}{x^4}}=0$$

$$\textbf{g)}\lim_{x\to1}\frac{\sqrt[4]{x^3-x}}{\sqrt{x^2+x-2}}=\left(\frac{0}{0}\right)=\lim_{x\to1}\frac{\sqrt[4]{x\cdot(x-1)\cdot(x+1)}}{\sqrt{(x-1)\cdot(x+2)}}=\lim_{x\to1}\sqrt[4]{\frac{x\cdot\cancel{(x-1)}\cdot(x+1)}{(x-1)^{\cancel{2}}\cdot(x+2)^2}}=$$

$$=\lim_{x\to1}\sqrt[4]{\frac{x\cdot(x+1)}{(x-1)\cdot(x+2)^2}}=\left(\frac{k}{0}\right)=\left\langle\begin{array}{l}\lim\limits_{x\to1^+}\sqrt[4]{\dfrac{x\cdot(x+1)}{(x-1)\cdot(x+2)^2}}=+\infty\\ \lim\limits_{x\to1^-}\sqrt[4]{\dfrac{x\cdot(x+1)}{(x-1)\cdot(x+2)^2}}=\text{No existe}\end{array}\right.$$

$$\textbf{h)}\ \lim_{x\to 6}\left(\frac{x^2-4x-10}{x-4}\right)^{\frac{1}{x-6}}=(1)^{\left(\frac{1}{0}\right)}=\left\{\begin{matrix}\text{límites}\\ \text{laterales}\end{matrix}\right\}=\left\langle\begin{matrix}\lim\limits_{x\to 6^+}\left(\dfrac{x^2-4x-10}{x-4}\right)^{\frac{1}{x-6}}=\left(1^{+\infty}\right)=\\ \lim\limits_{x\to 6^-}\left(\dfrac{x^2-4x-10}{x-4}\right)^{\frac{1}{x-6}}=\left(1^{-\infty}\right)=0\end{matrix}\right.=$$

$$=\left\langle=e^{\lim\limits_{x\to 6^+}\frac{1}{x-6}\cdot\left(\frac{x^2-4x-10}{x-4}-1\right)}=e^{\lim\limits_{x\to 6^+}\frac{1}{x-6}\cdot\frac{x^2-5x-6}{x-4}}=e^{\lim\limits_{x\to 6^+}\frac{(x+1)\cdot\cancel{(x-6)}}{(x-4)\cdot\cancel{(x-6)}}}=e^{\lim\limits_{x\to 6^+}\frac{x+1}{x-4}}=e^{7/2}\right.$$

$$\textbf{i)}\ \lim_{x\to\frac{3\pi}{2}}(1+\operatorname{sen}x)\cdot\operatorname{tg}^2 x=(0\cdot\infty)=\lim_{x\to\frac{3\pi}{2}}\frac{(1+\operatorname{sen}x)\cdot\operatorname{sen}^2 x}{\cos^2 x}=\left(\frac{0}{0}\right)=\lim_{x\to\frac{3\pi}{2}}\frac{(1+\operatorname{sen}x)\cdot\operatorname{sen}^2 x}{1-\operatorname{sen}^2 x}=$$

$$\lim_{x\to\frac{3\pi}{2}}\frac{\cancel{(1+\operatorname{sen}x)}\cdot\operatorname{sen}^2 x}{(1-\operatorname{sen}x)\cdot\cancel{(1+\operatorname{sen}x)}}=\lim_{x\to\frac{3\pi}{2}}\frac{\operatorname{sen}^2 x}{(1-\operatorname{sen}x)}=\frac{1}{2}$$

$$\textbf{j)}\ \lim_{x\to 0}\frac{\sqrt{1-\cos x}}{x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\sqrt{1-\cos x}\cdot\sqrt{1+\cos x}}{x\cdot\sqrt{1+\cos x}}=\lim_{x\to 0}\frac{\sqrt{1-\cos^2 x}}{x\cdot\sqrt{1+\cos x}}=\lim_{x\to 0}\frac{\sqrt{\operatorname{sen}^2 x}}{x\cdot\sqrt{1+\cos x}}=$$

$$\underbrace{=\!=\!=\!=\!=\!=\!=\!=\!=\!=}_{\lim\limits_{x\to 0}\operatorname{sen}x=\lim\limits_{x\to 0}x}\lim_{x\to 0}\frac{\cancel{x}}{\cancel{x}\cdot\sqrt{1+\cos x}}=\lim_{x\to 0}\frac{1}{\sqrt{1+\cos x}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$

$$\textbf{k)}\ \lim_{x\to 0}\frac{\operatorname{sen}x}{x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{x}{x}=1$$

$$\textbf{l)}\ \lim_{x\to 0}\frac{1-\cos x}{x^2}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{(1-\cos x)\cdot(1+\cos x)}{x^2\cdot(1+\cos x)}=\lim_{x\to 0}\frac{1-\cos^2 x}{x^2\cdot(1+\cos x)}=\lim_{x\to 0}\frac{\operatorname{sen}^2 x}{x^2\cdot(1+\cos x)}=$$

$$=\lim_{x\to 0}\frac{\cancel{x^2}}{\cancel{x^2}\cdot(1+\cos x)}=\lim_{x\to 0}\frac{1}{1+\cos x}=\frac{1}{2}$$

m) $\lim\limits_{x\to -1}\frac{\sqrt{2+x}-1}{\sqrt{8-x}-3}=\left(\frac{0}{0}\right)=\lim\limits_{x\to -1}\frac{\left(\sqrt{2+x}-1\right)\cdot\left(\sqrt{8-x}+3\right)}{\left(\sqrt{8-x}-3\right)\cdot\left(\sqrt{8-x}+3\right)}=\lim\limits_{x\to -1}\frac{\left(\sqrt{2+x}-1\right)\cdot\left(\sqrt{8-x}+3\right)}{8-x-9}=$

$=\lim\limits_{x\to -1}\frac{\left(\sqrt{2+x}-1\right)\cdot\left(\sqrt{8-x}+3\right)}{-x-1}=\left(\frac{0}{0}\right)=\lim\limits_{x\to -1}\frac{\left(\sqrt{2+x}-1\right)\cdot\left(\sqrt{2+x}+1\right)\cdot\left(\sqrt{8-x}+3\right)}{(-x-1)\cdot\left(\sqrt{2+x}+1\right)}=$

$=\lim\limits_{x\to -1}\frac{(2+x-1)\cdot\left(\sqrt{8-x}+3\right)}{(-x-1)\cdot\left(\sqrt{2+x}+1\right)}=\lim\limits_{x\to -1}\frac{(x+1)\cdot\left(\sqrt{8-x}+3\right)}{(-x-1)\cdot\left(\sqrt{2+x}+1\right)}=\lim\limits_{x\to -1}\frac{\cancel{(x+1)}\cdot\left(\sqrt{8-x}+3\right)}{-\cancel{(x+1)}\cdot\left(\sqrt{2+x}+1\right)}=$

$=\lim\limits_{x\to -1}\frac{\left(\sqrt{8-x}+3\right)}{-\left(\sqrt{2+x}+1\right)}=\frac{3+3}{-(1+1)}=-3$

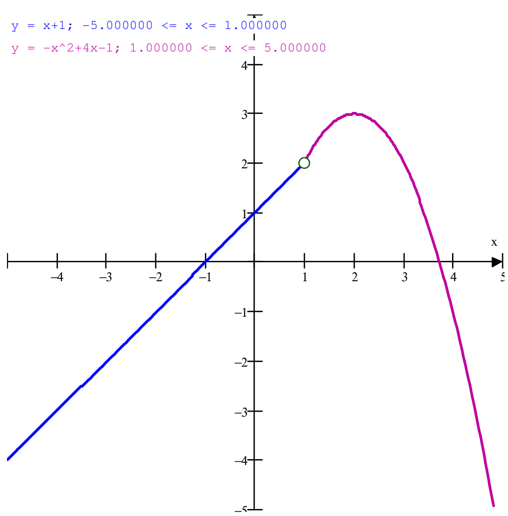
LÍMITES LATERALES

LyC-10 Calcular si existe $\underset{x\to 1}{lím}\, f(x)$, siendo: $f(x)=\begin{cases} x+1 & x<1 \\ -x^2+4x-1 & x>1 \end{cases}$

Solución:

$$\lim_{x\to 1} f(x) = \left\langle \begin{array}{l} \underset{x\to 1^+}{\text{lím}}\ f(x) = \underset{x\to 1^+}{\text{lím}}\left(x+1\right) = 2 \\ \underset{x\to 1^-}{\text{lím}}\ f(x) = \underset{x\to 1^-}{\text{lím}}\left(-x^2+4x-1\right) = 2 \end{array} \right.$$

Como los límites laterales coinciden, existe el $\underset{x\to 1}{lím}\, f(x)$ y es igual a 2



L'HÔPITAL $x \to k$

INDETER. $\left(\frac{0}{0}\right)$ y $\left(\frac{\infty}{\infty}\right)$

LyC-11 Calcula los siguientes límites:

a) $\lim\limits_{x\to 0} \dfrac{(3-x)\cdot e^x - 2x - 3}{x^2}$

b) $\lim\limits_{x\to 0} \dfrac{\operatorname{sen} x\cdot(1+\cos x)}{x\cdot \cos x}$

c) $\lim\limits_{x\to 0} \dfrac{e^x - e^{-x}}{\operatorname{sen} x}$

d) $\lim\limits_{x\to 0} \dfrac{e^{-x} + x - 1}{x^2}$

e) $\lim\limits_{x\to 0} \dfrac{x^3 + 2x^2 + x}{x^3 + x^2 - x - 1}$

f) $\lim\limits_{x\to +\infty} \dfrac{3^x}{x^3}$

g) $\lim\limits_{x\to +\infty} x\left(5^{1/x} - 1\right)$

h) $\lim\limits_{x\to 0} (\cos 2x)^{3/x^2}$

i) $\lim\limits_{x\to 0} (\cos x + \operatorname{sen} x)^{1/x}$

j) $\lim\limits_{x\to +\infty} \left(1 - 2^{1/x}\right)\cdot x$

k) $\lim\limits_{x\to 1} \left[\dfrac{1}{x-1} - \dfrac{1}{\ln x}\right]$

l) $\lim\limits_{x\to 0} \dfrac{x\cdot \cos x - \operatorname{sen} x}{x^3}$

m) $\lim\limits_{x\to 0} \dfrac{x - \operatorname{sen} x}{x^3}$

n) $\lim\limits_{x\to 0} \left(\dfrac{1}{x^2} - \dfrac{1}{\operatorname{sen}^2 x}\right)$

o) $\lim\limits_{x\to 1^+} \left(\dfrac{2x}{x-1} - \dfrac{3x}{\ln x}\right)$

p) $\lim\limits_{x\to 0} \dfrac{(2-x)\cdot e^x - (2+x)}{x^2}$

Solución:

a) $$\lim_{x\to 0} \frac{(3-x)e^x - 2x - 3}{x^2} = \left(\frac{0}{0}\right) \overbrace{====}^{L'Hôpital} \lim_{x\to 0} \frac{-e^x + (3-x)e^x - 2}{2x} = \lim_{x\to 0} \frac{(2-x)e^x - 2}{2x} = \left(\frac{0}{0}\right) \overbrace{====}^{L'Hôpital}$$
$$= \frac{-e^x + (2-x)e^x}{2} = \lim_{x\to 0} \frac{(1-x)e^x}{2} = \frac{1}{2}$$

b) $$\lim_{x\to 0} \frac{\operatorname{sen} x\cdot(1+\cos x)}{x\cdot \cos x} = \left(\frac{0}{0}\right) \overbrace{====}^{L'Hôpital} \lim_{x\to 0} \frac{\cos x\cdot(1+\cos x) - \operatorname{sen}^2 x}{\cos x - x\cdot \operatorname{sen} x} = \frac{1+1-0}{1-0} = 2$$

$$\textbf{c)}\lim_{x\to 0}\frac{e^{x}-e^{-x}}{\operatorname{sen}x}=\left(\frac{0}{0}\right)\overbrace{=\!=\!=\!=}^{L'Hôpital}\lim_{x\to 0}\frac{e^{x}+e^{-x}}{\cos x}=\frac{1+1}{1}=2$$

$$\textbf{d)}\lim_{x\to 0}\frac{e^{-x}+x-1}{x^{2}}=\left(\frac{0}{0}\right)\overbrace{=\!=\!=\!=}^{L'Hôpital}\lim_{x\to 0}\frac{-e^{x}+1}{2x}=\left(\frac{0}{0}\right)\overbrace{=\!=\!=\!=}^{L'Hôpital}\lim_{x\to 0}\frac{-e^{x}}{2}=\frac{-1}{2}$$

$$\textbf{e)}\lim_{x\to 0}\frac{x^{3}+2x^{2}+x}{x^{3}+x^{2}-x-1}=\left(\frac{0}{0}\right)\overbrace{=\!=\!=\!=}^{L'Hôpital}\lim_{x\to 0}\frac{3x^{2}+4x+1}{3x^{2}+2x-1}=-1$$

$$\textbf{f)}\lim_{x\to +\infty}\frac{3^{x}}{x^{3}}=\left(\frac{\infty}{\infty}\right)\overbrace{=\!=\!=\!=}^{L'Hôpital}\lim_{x\to +\infty}\frac{3^{x}\cdot \ln 3}{3x^{2}}=\left(\frac{\infty}{\infty}\right)\overbrace{=\!=\!=\!=}^{L'Hôpital}\lim_{x\to +\infty}\frac{3^{x}\cdot(\ln 3)^{2}}{6x}=\left(\frac{\infty}{\infty}\right)\overbrace{=\!=\!=\!=}^{L'Hôpital}$$

$$=\lim_{x\to +\infty}\frac{3^{x}\cdot(\ln 3)^{3}}{6}=+\infty$$

$$\textbf{g)}\lim_{x\to +\infty}x\cdot\left(5^{1/x}-1\right)=\lim_{x\to +\infty}\frac{5^{1/x}-1}{1/x}=\left(\frac{0}{0}\right)\overbrace{=\!=\!=\!=}^{L'Hôpital}\lim_{x\to +\infty}\frac{5^{1/x}\cdot\ln 5\cdot\cancel{\left(\frac{-1}{x^{2}}\right)}}{\cancel{\frac{-1}{x^{2}}}}=\lim_{x\to +\infty}5^{1/x}\cdot\ln 5=\ln 5$$

$$\textbf{h)}\lim_{x\to 0}(\cos 2x)^{3/x^{2}}=\left(1^{\infty}\right)=e^{\lim\limits_{x\to 0}\frac{3}{x^{2}}\cdot(\cos 2x-1)}=e^{(\infty\cdot 0)}=e^{\lim\limits_{x\to 0}\frac{3\cdot(\cos 2x-1)}{x^{2}}}=e^{\left(\frac{0}{0}\right)}\overbrace{=\!=\!=\!=}^{L'Hôpital}e^{\lim\limits_{x\to 0}\frac{-6\operatorname{sen}2x}{2x}}=$$

$$=e^{\left(\frac{0}{0}\right)}\overbrace{=\!=\!=\!=}^{L'Hôpital}e^{\lim\limits_{x\to 0}\frac{-12\cos 2x}{2}}=e^{-6}$$

$$\textbf{i)}\lim_{x\to 0}(\cos x+\operatorname{sen}x)^{1/x}=\left(1^{\infty}\right)=e^{\lim\limits_{x\to 0}\frac{1}{x}\cdot(\cos x+\operatorname{sen}x-1)}=e^{\lim\limits_{x\to 0}\frac{\cos x+\operatorname{sen}x-1}{x}}=e^{\left(\frac{0}{0}\right)}\overbrace{=\!=\!=\!=}^{L'Hôpital}e^{\lim\limits_{x\to 0}\frac{-\operatorname{sen}x+\cos x}{1}}=e$$

$$\textbf{j)}\lim_{x\to +\infty}\left(1-2^{1/x}\right)\cdot x=(0\cdot\infty)=\lim_{x\to +\infty}\frac{1-2^{1/x}}{1/x}=\left(\frac{0}{0}\right)\overbrace{=\!=\!=\!=}^{L'Hôpital}\lim_{x\to +\infty}\frac{-2^{1/x}\cdot\ln 2\cdot\cancel{\left(-\frac{1}{x^{2}}\right)}}{\cancel{-\frac{1}{x^{2}}}}=$$

$$=\lim_{x\to +\infty}\left(-2^{1/x}\cdot\ln 2\right)=-\ln 2=\ln(1/2)$$

$$\mathbf{k)} \lim_{x\to 1}\left[\frac{1}{x-1}-\frac{1}{\ln x}\right]=(\infty-\infty)=\lim_{x\to 1}\frac{\ln x - x+1}{(x-1)\cdot \ln x}=\left(\frac{0}{0}\right)\overbrace{=\!=\!=\!=}^{L'Hôpital}\lim_{x\to 1}\frac{\frac{1}{x}-1}{\ln x+\frac{x-1}{x}}=$$

$$=\lim_{x\to 1}\frac{\frac{1-x}{\cancel{x}}}{\frac{x\cdot \ln x + x-1}{\cancel{x}}}=\lim_{x\to 1}\frac{1-x}{x\cdot \ln x+x-1}=\left(\frac{0}{0}\right)\overbrace{=\!=\!=\!=}^{L'Hôpital}\lim_{x\to 1}\frac{-1}{\ln x+\cancel{x}\cdot\frac{1}{\cancel{x}}+1}=$$

$$=\lim_{x\to 1}\frac{-1}{\ln x+2}=\frac{-1}{2}$$

$$\mathbf{l)} \lim_{x\to 0}\frac{x\cdot \cos x-\operatorname{sen} x}{x^3}=\left(\frac{0}{0}\right)\overbrace{=\!=\!=\!=}^{L'Hôpital}\lim_{x\to 0}\frac{\cancel{\cos x}-x\cdot \operatorname{sen} x-\cancel{\cos x}}{3x^2}=\left(\frac{0}{0}\right)\overbrace{=\!=\!=\!=}^{L'Hôpital}$$

$$=\lim_{x\to 0}\frac{-\operatorname{sen} x-x\cdot\cos x}{6x}=\left(\frac{0}{0}\right)\overbrace{=\!=\!=\!=}^{L'Hôpital}\lim_{x\to 0}\frac{-\cos x-\cos x+x\cdot \operatorname{sen} x}{6}=$$

$$=\lim_{x\to 0}\frac{-2\cos x+x\cdot\operatorname{sen} x}{6}=-\frac{2}{6}=-\frac{1}{3}$$

$$\mathbf{m)} \lim_{x\to 0}\frac{x-\operatorname{sen} x}{x^3}=\lim_{x\to 0}\frac{x-\operatorname{sen} x}{x^3}=\left(\frac{0}{0}\right)\overbrace{=\!=\!=\!=}^{L'Hôpital}\lim_{x\to 0}\frac{1-\cos x}{3x^2}=\left(\frac{0}{0}\right)\overbrace{=\!=\!=\!=}^{L'Hôpital}\lim_{x\to 0}\frac{\operatorname{sen} x}{6x}=\left(\frac{0}{0}\right)\overbrace{=\!=\!=\!=}^{L'Hôpital}$$

$$=\lim_{x\to 0}\frac{\cos x}{6}=\frac{1}{6}$$

$$\textbf{n)}\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\operatorname{sen}^2 x}\right)=(\infty-\infty)=\lim_{x\to 0}\frac{\operatorname{sen}^2 x-x^2}{x^2\cdot\operatorname{sen}^2 x}=\left(\frac{0}{0}\right)\overbrace{=}^{L'Hôp}\lim_{x\to 0}\frac{2\operatorname{sen} x\cdot\cos x-2x}{2x\cdot\operatorname{sen}^2 x+2x^2\cdot\operatorname{sen} x\cdot\cos x}=$$

$$=\lim_{x\to 0}\frac{\operatorname{sen} 2x-2x}{2x\cdot\operatorname{sen}^2 x+x^2\cdot\operatorname{sen} 2x}=\left(\frac{0}{0}\right)\overbrace{=====}^{L'Hôpital}$$

$$=\lim_{x\to 0}\frac{2\cos 2x-2}{2\cdot\operatorname{sen}^2 x+4x\cdot\operatorname{sen} x\cdot\cos x+2x\cdot\operatorname{sen} 2x+2x^2\cdot\cos 2x}\overbrace{=========}^{4x\cdot\operatorname{sen} x\cdot\cos x=2x\cdot\operatorname{sen} 2x}$$

$$=\lim_{x\to 0}\frac{2\cos 2x-2}{2\cdot\operatorname{sen}^2 x+4x\cdot\operatorname{sen} 2x+2x^2\cdot\cos 2x}=\left(\frac{0}{0}\right)\overbrace{=====}^{L'Hôpital}$$

$$=\lim_{x\to 0}\frac{-4\operatorname{sen} 2x}{4\cdot\operatorname{sen} x\cdot\cos x+4\cdot\operatorname{sen} 2x+8x\cdot\cos 2x+4x\cdot\cos 2x-4x^2\operatorname{sen} 2x}=$$

$$=\lim_{x\to 0}\frac{-4\operatorname{sen} 2x}{6\cdot\operatorname{sen} 2x+12x\cdot\cos 2x-4x^2\cdot\operatorname{sen} 2x}=\left(\frac{0}{0}\right)\overbrace{=====}^{L'Hôpital}$$

$$=\lim_{x\to 0}\frac{-8\cos 2x}{12\cdot\cos 2x+12\cdot\cos 2x-24x\cdot\operatorname{sen} 2x-8x\cdot\operatorname{sen} 2x-8x^2\cdot\cos 2x}=$$

$$=\lim_{x\to 0}\frac{-8\cos 2x}{24\cdot\cos 2x-32x\cdot\operatorname{sen} 2x-8x^2\cdot\cos 2x}=-\frac{8}{24}=-\frac{1}{3}$$

$$\textbf{o)}\lim_{x\to 1^+}\left(\frac{2x}{x-1}-\frac{3x}{\ln x}\right)=(\infty-\infty)=\lim_{x\to 1^+}\left(\frac{2x\cdot\ln x-3x^2+3x}{(x-1)\cdot\ln x}\right)=\left(\frac{0}{0}\right)\overbrace{=====}^{L'Hôpital}$$

$$=\lim_{x\to 1^+}\left(\frac{2\ln x+2-6x+3}{\ln x+\frac{x-1}{x}}\right)=-\infty$$

$$\textbf{p)}\lim_{x\to 0}\frac{(2-x)\cdot e^x-(2+x)}{x^2}=\left(\frac{0}{0}\right)\overbrace{=====}^{L'Hôpital}\lim_{x\to 0}\frac{-e^x+(2-x)\cdot e^x-1}{2x}=\lim_{x\to 0}\frac{(-1+2-x)\cdot e^x-1}{2x}=$$

$$=\lim_{x\to 0}\frac{(1-x)\cdot e^x-1}{2x}=\left(\frac{0}{0}\right)\overbrace{=====}^{L'Hôpital}\lim_{x\to 0}\frac{-e^x+(1-x)\cdot e^x}{2}=$$

$$=\lim_{x\to 0}\frac{(-1+1-x)\cdot e^x}{2}\lim_{x\to 0}\frac{-x\cdot e^x}{2}=0$$

NEPERIANOS

INDET. $\left(\infty^0\right)$ $\left(1^\infty\right)$ $\left(0^0\right)$

LyC-12 Calcula el valor de los siguientes límites:

a) $\lim_{x\to 0}\left(\frac{1}{x}\right)^{\operatorname{sen} x}$

b) $\lim_{x\to 0}\left(\cos x+\operatorname{sen} x\right)^{1/x}$

c) $\lim_{x\to \pi/2}\left(\operatorname{tg} x\right)^{\cos x}$

d) $\lim_{x\to 0}\left(x^2-2x\right)^{x^2}$

e) $\lim_{x\to 1}\left(x\right)^{\frac{1}{1-x}}$

f) $\boxed{\lim_{x\to 0}\left(1+4x\right)^{32/x^3}}$

g) $\boxed{\lim_{x\to +\infty}\left(\frac{x^2-3}{x^2+3}\right)^{ax^2}}$

h) $\lim_{x\to \pi/2}\left(1+2\cos x\right)^{1/\cos x}$

Solución:

a) $\lim_{x\to 0}\left(\frac{1}{x}\right)^{\operatorname{sen} x}=\left(\infty^0\right)\ \to\ L=\lim_{x\to 0}\left(\frac{1}{x}\right)^{\operatorname{sen} x}\xrightarrow[\text{neperianos}]{\text{Tomando}}$

$$\ln L=\ln\left[\lim_{x\to 0}\left(\frac{1}{x}\right)^{\operatorname{sen} x}\right]=\lim_{x\to 0}\left[\ln\left(\frac{1}{x}\right)^{\operatorname{sen} x}\right]=\lim_{x\to 0}\left[\operatorname{sen} x\cdot\ln\left(\frac{1}{x}\right)\right]=\left(0\cdot\infty\right)=\lim_{x\to 0}\frac{\ln\frac{1}{x}}{\frac{1}{\operatorname{sen} x}}=$$

$$=\left(\frac{\infty}{\infty}\right)\overset{L'H\hat{o}pital}{\overbrace{====}}\lim_{x\to 0}\frac{x\cdot\left(\frac{-1}{x^2}\right)}{-\frac{1}{\operatorname{sen}^2 x}\cdot\cos x}=\lim_{x\to 0}\frac{\operatorname{sen}^2 x}{x\cdot\cos x}=\left(\frac{0}{0}\right)\overset{L'H\hat{o}pital}{\overbrace{====}}\lim_{x\to 0}\frac{2\operatorname{sen} x\cdot\cos x}{\cos x-x\cdot\operatorname{sen} x}=\frac{0}{1}=0\to$$

$$\to\quad L=e^0=1$$

b) $\lim\limits_{x\to 0}\left(\cos x+\operatorname{sen}x\right)^{1/x}=\left(1^{\infty}\right)\quad\rightarrow\quad L=\lim\limits_{x\to 0}\left(\cos x+\operatorname{sen}x\right)^{1/x}\xrightarrow[\text{neperianos}]{\text{Tomando}}$

$$\ln L=\lim_{x\to 0}\left[\frac{1}{x}\cdot\ln\left(\cos x+\operatorname{sen}x\right)\right]=\left(\frac{\infty}{0}\right)=\lim_{x\to 0}\left[\frac{\ln\left(\cos x+\operatorname{sen}x\right)}{x}\right]=\left(\frac{0}{0}\right)\overbrace{====}^{L'Hôpital}$$

$$=\lim_{x\to 0}\left[\frac{\dfrac{-\operatorname{sen}x+\cos x}{\cos x+\operatorname{sen}x}}{1}\right]=1\quad\rightarrow\quad L=e^{1}=e$$

c) $\lim\limits_{x\to\pi/2}\left(\operatorname{tg}x\right)^{\cos x}=\left(\infty^{0}\right)\quad\rightarrow\quad L=\lim\limits_{x\to\pi/2}\left(\operatorname{tg}x\right)^{\cos x}\xrightarrow[\text{neperianos}]{\text{Tomando}}$

$$\ln L=\lim_{x\to 0}\left[\cos x\cdot\ln\left(\operatorname{tg}x\right)\right]=\left(0\cdot\infty\right)=\lim_{x\to 0}\frac{\ln\left(\operatorname{tg}x\right)}{1/\cos x}=\left(\frac{\infty}{\infty}\right)\overbrace{=====}^{L'Hôpital}\lim_{x\to 0}\frac{\dfrac{\sec^{2}x}{\operatorname{tg}x}}{\dfrac{\operatorname{sen}x}{\cos^{2}x}}=$$

$$=\lim_{x\to 0}\left[\frac{\cos x}{\cancel{\cos^{2}x}\cdot\operatorname{sen}x}\cdot\frac{\cancel{\cos^{2}x}}{\operatorname{sen}x}\right]=\left(\frac{\infty}{\infty}\right)\overbrace{=====}^{L'Hôpital}=\lim_{x\to 0}\frac{\cos x}{\operatorname{sen}^{2}x}=0\rightarrow L=e^{0}=1$$

d) $\lim\limits_{x\to 0}\left(x^{2}-2x\right)^{x^{2}}=\left(0^{0}\right)\quad\rightarrow\quad L=\lim\limits_{x\to 0}\left(x^{2}-2x\right)^{x^{2}}\xrightarrow[\text{neperianos}]{\text{Tomando}}$

$$\ln L=\lim_{x\to 0}\left[x^{2}\cdot\ln\left(x^{2}-2x\right)\right]=\left(0\cdot\infty\right)=\lim_{x\to 0}\frac{\ln\left(x^{2}-2x\right)}{1/x^{2}}=\lim_{x\to 0}\frac{\dfrac{2x-2}{x^{2}-2x}}{\dfrac{-2}{x^{3}}}\overbrace{=====}^{Factorizamos}$$

$$=\lim_{x\to 0}\frac{\cancel{2}x^{\cancel{3}2}\cdot\left(x-1\right)}{\cancel{2}\,\cancel{x}\cdot\left(x-1\right)}=0\quad\rightarrow\quad L=e^{0}=1$$

e) $\lim\limits_{x\to 1}\left(x\right)^{\frac{1}{1-x}}=\left(1^{\infty}\right)\quad\rightarrow\quad L=\lim\limits_{x\to 1}\left(x\right)^{\frac{1}{1-x}}\xrightarrow[\text{neperianos}]{\text{Tomando}}$

$$\ln L=\lim_{x\to 1}\left(\frac{1}{1-x}\cdot\ln x\right)=\left(\infty\cdot 0\right)=\lim_{x\to 1}\frac{\ln x}{1-x}=\left(\frac{\infty}{\infty}\right)\overbrace{=====}^{L'Hôpital}\lim_{x\to 1}\frac{1/x}{-1}\lim_{x\to 1}\frac{-1}{x}=-1$$

$$\rightarrow\quad L=e^{-1}=\frac{1}{e}$$

f) $\lim\limits_{x\to 0}(1+4x)^{32/x^3}=(1^\infty)=L \xrightarrow{\text{Tomando logaritmos neperianos}}$

$$\ln L=\ln\left[\lim_{x\to 0}(1+4x)^{32/x^3}\right]=\lim_{x\to 0}\left[\ln(1+4x)^{32/x^3}\right]=\lim_{x\to 0}\left[\frac{32}{x^3}\cdot\ln(1+4x)\right]=$$

$$=\lim_{x\to 0}\frac{32\cdot\ln(1+4x)}{x^3}=\left(\frac{0}{0}\right)\overset{\overbrace{L'Hôpital}}{=\!=\!=\!=\!=}\lim_{x\to 0}\frac{32\cdot\dfrac{4}{1+4x}}{3x^2}=\lim_{x\to 0}\frac{128}{3x^2\cdot(1+4x)}=\left(\frac{k}{0}\right)=$$

$$=\left\{\begin{array}{l}\lim\limits_{x\to 0^+}\dfrac{128}{3x^2\cdot(1+4x)}=+\infty\\ \lim\limits_{x\to 0^-}\dfrac{128}{3x^2\cdot(1+4x)}=+\infty\end{array}\right\}=+\infty \quad\to\quad L=e^{+\infty}=+\infty$$

g) $\lim\limits_{x\to+\infty}\left(\dfrac{x^2-3}{x^2+3}\right)^{ax^2}=(1^\infty)\xrightarrow{\text{Tomando neperianos}}$

$$\ln L=\ln\left[\lim_{x\to+\infty}\left(\frac{x^2-3}{x^2+3}\right)^{ax^2}\right]=\lim_{x\to+\infty}\left[\ln\left(\frac{x^2-3}{x^2+3}\right)^{ax^2}\right]=\lim_{x\to+\infty}\left[ax^2\cdot\ln\left(\frac{x^2-3}{x^2+3}\right)\right]=$$

$$=(\infty\cdot 0)=\lim_{x\to+\infty}\left[\frac{\ln\left(\dfrac{x^2-3}{x^2+3}\right)}{\dfrac{1}{ax^2}}\right]=\left(\frac{0}{0}\right)\overset{\overbrace{L'Hôpital}}{=\!=\!=}\lim_{x\to+\infty}\left[\frac{\ln\left(\dfrac{2x\cdot(x^2+3)-(x^2-3)\cdot 2x}{(x^2+3)^2}\right)}{\dfrac{1}{ax^2}}\right]=$$

$$=\lim_{x\to+\infty}\left[\frac{\dfrac{12x}{(x^2+3)^2}}{\dfrac{-2}{ax^3}}\right]=\lim_{x\to+\infty}\left[\frac{12ax^4}{-2x^4-12x-18}\right]=-6a\quad\to\quad L=e^{-6a}=\frac{1}{e^{6a}}$$

h) $\lim\limits_{x\to\pi/2}(1+2\cos x)^{1/\cos x}=(1^\infty)\quad\to\quad L=\lim\limits_{x\to\pi/2}(1+2\cos x)^{1/\cos x}\xrightarrow{\text{Tomando neperianos}}$

$$\ln L=\lim_{x\to\pi/2}\left[\frac{1}{\cos x}\cdot\ln(1+2\cos x)\right]=(\infty\cdot 0)=\lim_{x\to\pi/2}\frac{\ln(1+2\cos x)}{\cos x}\overset{\overbrace{L^*Hôpital}}{=\!=\!=\!=\!=}$$

$$=\lim_{x\to\pi/2}\frac{\dfrac{-2\cancel{\operatorname{sen}x}}{1+2\cos x}}{-\cancel{\operatorname{sen}x}}=\lim_{x\to\pi}\frac{2}{1+2\cos x}=2\quad\to\quad L=e^2$$

LÍMITES CON VALOR ABSOLUTO

LyC-13 Calcula los sigientes límites:

a) $\lim\limits_{x\to+\infty}\dfrac{1+3x}{|x-1|+2}$ b) $\lim\limits_{x\to-\infty}\dfrac{1+3x}{|x-1|+2}$ c) $\lim\limits_{x\to1}\dfrac{1+3x}{|x-1|+2}$

d) $\lim\limits_{x\to+\infty}\left(|2x+4|-|x-1|\right)$ e) $\lim\limits_{x\to-\infty}\left(|2x+4|-|x-1|\right)$

f) $\lim\limits_{x\to-2}\left(|2x+4|-|x-1|\right)$ g) $\lim\limits_{x\to1}\left(|2x+4|-|x-1|\right)$

h) $\lim\limits_{x\to+\infty}\dfrac{|x-3|}{|x|-3}$ i) $\lim\limits_{x\to-\infty}\dfrac{|x-3|}{|x|-3}$

j) $\lim\limits_{x\to0}\dfrac{|x-3|}{|x|-3}$ k) $\lim\limits_{x\to3}\dfrac{|x-3|}{|x|-3}$

l) $\lim\limits_{x\to-\infty}\dfrac{|x-3|}{|x+1|}$ m) $\lim\limits_{x\to+\infty}\dfrac{|x-3|}{|x+1|}$

n) $\lim\limits_{x\to-1}\dfrac{|x-3|}{|x+1|}$ o) $\lim\limits_{x\to3}\dfrac{|x-3|}{|x+1|}$

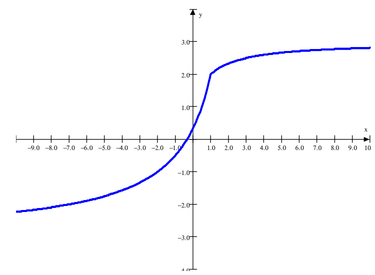
Solución:

Cuando tenemos un valor absoluto en un límite tenemos que expresar la función como una función “a trozos”:

$$f(x)=\begin{cases}\dfrac{1+3x}{-(x-1)+2} & si \quad x-1<0\\[2mm] \dfrac{1+3x}{x-1+2} & si \quad x-1\geq0\end{cases} \rightarrow f(x)=\begin{cases}\dfrac{1+3x}{3-x} & si \quad x<1\\[2mm] \dfrac{1+3x}{x+1} & si \quad x\geq1\end{cases}$$

a) $\lim\limits_{x\to+\infty}\dfrac{1+3x}{|x-1|+2}=\lim\limits_{x\to+\infty}\dfrac{1+3x}{x+1}=3$

b) $\lim\limits_{x\to-\infty}\dfrac{1+3x}{|x-1|+2}=\lim\limits_{x\to-\infty}\dfrac{1+3x}{3-x}=\lim\limits_{x\to+\infty}\dfrac{1-3x}{3+x}=-3$



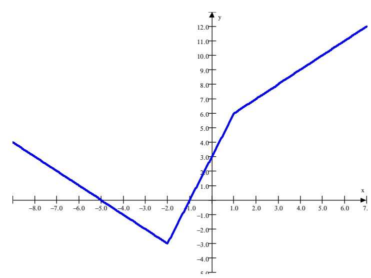
c) $\lim\limits_{x\to1}\dfrac{1+3x}{|x-1|+2}=\left\{\begin{array}{l}\lim\limits_{x\to1^-}\dfrac{1+3x}{3-x}=2\\ \lim\limits_{x\to1^+}\dfrac{1+3x}{x+1}\end{array}\right\}=2$

Analizamos los siguientes valores absolutos independientemente y luego en conjunto.

$$|x-1|=\begin{cases}-x+1 & si \quad x<1\\ x-1 & si \quad x\geq1\end{cases}\qquad |2x+4|=\begin{cases}-2x-4 & si \quad x<-2\\ 2x+4 & si \quad x\geq-2\end{cases}$$

$$f(x)=\begin{cases}-2x-4-(-x+1) & si \quad x<-2\\ 2x+4-(-x+1) & si \quad -2\leq x<1\\ 2x+4-(x-1) & si \quad x\geq1\end{cases}\Rightarrow f(x)=\begin{cases}-x-5 & si \quad x<-2\\ 3x+3 & si \quad -2\leq x<1\\ x+5 & si \quad x\geq1\end{cases}$$

d) $\lim\limits_{x\to+\infty}\left(|2x+4|-|x-1|\right)=\lim\limits_{x\to+\infty}(x+5)=+\infty$



e) $\lim\limits_{x\to-\infty}\left(|2x+4|-|x-1|\right)=\lim\limits_{x\to-\infty}(-x-5)=\lim\limits_{x\to+\infty}(x-5)=+\infty$

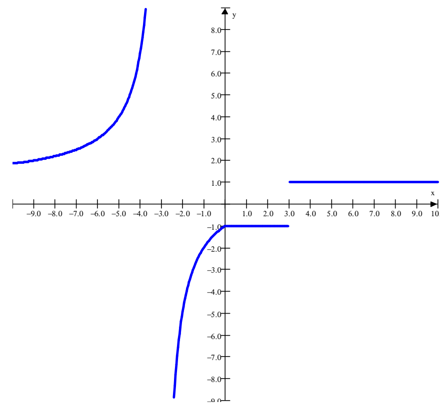
f) $\lim\limits_{x\to-2}\left(|2x+4|-|x-1|\right)=\left\{\begin{array}{l}\lim\limits_{x\to-2^-}(-x-5)=-3\\ \lim\limits_{x\to-2^+}(3x+3)=-3\end{array}\right\}=-3$

g) $\lim\limits_{x\to1}\left(|2x+4|-|x-1|\right)=\left\{\begin{array}{l}\lim\limits_{x\to1^-}(3x+3)=6\\ \lim\limits_{x\to1^+}(x+5)=6\end{array}\right\}=6$

$$|x-3|=\begin{cases}-x+3 & si \quad x<3\\ x-3 & si \quad x\geq3\end{cases}\qquad |x|=\begin{cases}-x & si \quad x<0\\ x & si \quad x\geq0\end{cases}$$

$$f(x)=\begin{cases}\dfrac{-x+3}{-x-3} & si \quad x<0\\ \dfrac{-x+3}{x-3} & si \quad 0\leq x<3\\ \dfrac{x-3}{x-3} & si \quad x\geq3\end{cases}\Rightarrow f(x)=\begin{cases}\dfrac{-x+3}{-x-3} & si \quad x<0\\ -1 & si \quad 0\leq x<3\\ 1 & si \quad x\geq3\end{cases}$$

h) $\lim\limits_{x\to+\infty}\dfrac{|x-3|}{|x|-3}=\lim\limits_{x\to+\infty}(1)=1$



i) $\lim\limits_{x\to-\infty}\dfrac{|x-3|}{|x|-3}=\lim\limits_{x\to-\infty}\dfrac{-x+3}{-x-3}=\lim\limits_{x\to+\infty}\dfrac{x+3}{x-3}=1$

j) $\lim\limits_{x\to0}\dfrac{|x-3|}{|x|-3}=\begin{Bmatrix}\lim\limits_{x\to0^-}\dfrac{-x+3}{-x-3}=-1\\ \lim\limits_{x\to0^+}(-1)=-1\end{Bmatrix}=-1$

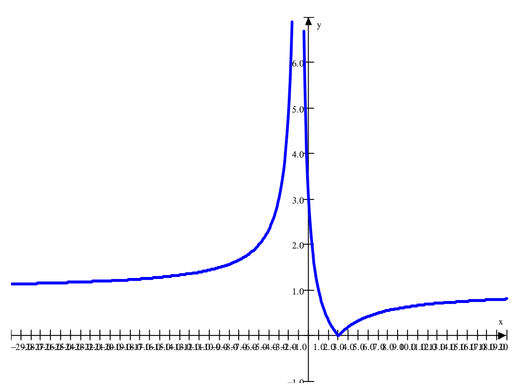
k) $\lim\limits_{x\to3}\dfrac{|x-3|}{|x|-3}=\begin{Bmatrix}\lim\limits_{x\to3^-}(-1)=-1\\ \lim\limits_{x\to3^+}(1)=1\end{Bmatrix}=\nexists$ límite

$$|x-3|=\begin{cases}-x+3 & si\quad x<3\\ x-3 & si\quad x\geq3\end{cases}\qquad |x+1|=\begin{cases}-x-1 & si\quad x<-1\\ x+1 & si\quad x\geq-1\end{cases}$$

$$f(x)=\begin{cases}\dfrac{-x+3}{-x-1} & si\quad x<-1\\ \dfrac{-x+3}{x+1} & si\quad -1\leq x<3\\ \dfrac{x-3}{x+1} & si\quad x\geq3\end{cases}\quad\Rightarrow\quad f(x)=\begin{cases}\dfrac{x-3}{x+1} & si\quad x<-1\\ \dfrac{-x+3}{x+1} & si\quad -1\leq x<3\\ \dfrac{x-3}{x+1} & si\quad x\geq3\end{cases}$$

l) $\lim\limits_{x\to-\infty}\dfrac{|x-3|}{|x+1|}=\lim\limits_{x\to-\infty}\dfrac{x-3}{x+1}=\lim\limits_{x\to+\infty}\dfrac{-x-3}{-x+1}=1$

m) $\lim\limits_{x\to+\infty}\dfrac{|x-3|}{|x+1|}=\lim\limits_{x\to+\infty}\dfrac{-x+3}{x+1}=-1$



n) $\lim\limits_{x\to-1}\dfrac{|x-3|}{|x+1|}=\left\langle\begin{matrix}\lim\limits_{x\to-1^-}\dfrac{x-3}{x+1}=+\infty\\ \lim\limits_{x\to-1^+}\dfrac{-x+3}{x+1}=+\infty\end{matrix}\right.$

o) $\lim\limits_{x\to3}\dfrac{|x-3|}{|x+1|}=\left\langle\begin{matrix}\lim\limits_{x\to3^-}\dfrac{-x+3}{x+1}=0\\ \lim\limits_{x\to3^+}\dfrac{x-3}{x+1}=0\end{matrix}\right.$

Solución:

Zzz Resolver y continuar metiendo desde los resueltos de la página 236
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